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JEE MAIN 2025

22 JANUARY SHIFT 1

MATHEMATICS SOLUTIONS

BY

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11th AND 12th STANDARD PCM FOR CBSE BOARD, JEE MAIN, NEET (UG), CUET AND OTHER ENTRANCE EXAMS

CLASS 11TH MATH

JEE-MAIN-2025 Math with Detailed solns. session-1 (22-1-2025)

Q.Nos

1. Sets : 17
2. Relations and functions : 15
3. Trigonometric functions :
4. CN and QEs:
   - QEs: 9
   - CN: 4
5. Linear Inequalities
6. Permutations and combinations 11, 22
7. Binomial Theorem
8. Sequence and series:
   - AP:
   - GP: 7
   - HP:
   - Misc Series: 10
9. Straight lines : 3
10. Conic Sections:
    - Circle: } 13-14
    - Parabola:
    - Ellipse:
    - Hyperbola: 1.
11. Introduction to 3D
12. Limits and Derivatives
13. Mathematical reasoning
14. Statistics
15. Probability

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CLASS 12TH MATH

JEE-MAIN-2025 Math with detailed solns session-1 (22-1-2025)

Q.Nos

1. Relations and functions : 20
2. Inverse Trigonometric functions : 5
3. Matrices } 23
4. Determinants
5. Continuity and Differentiability 2, 16, 21
6. Applications of Derivatives
7. Integrals :
8. Applications of integrals : 10
9. Differential Equations : 12
10. Vector Algebra : 25
11. Three-Dimensional Geometry 8, 24
12. Linear Programming
13. Probability : 6, 19

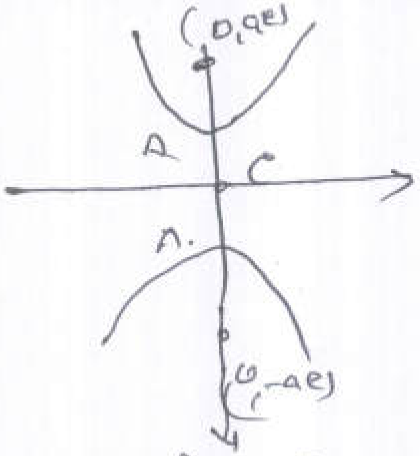
Q.1 **Let the foci of a hyperbola be (1, 14) and (1, -12). If it passes through the point (l, 6), then the length of its latus-rectum is:**

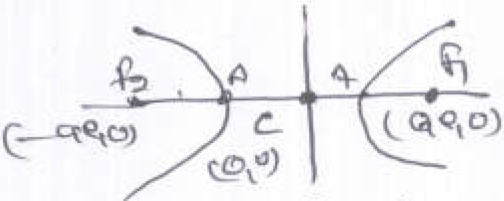
(1) $\frac{25}{6}$ (2) $\frac{24}{5}$

(3) $\frac{288}{5}$ (4) $\frac{144}{5}$

(JEE-Main 2025)

<u>CONCEPT APPLICABLE AND SOLUTION:</u>





$\underbrace{\frac{x^2}{a^2}-\frac{y^2}{b^2}=1}$

Nature Horizontal

$\frac{y^2}{a^2}-\frac{x^2}{b^2}=1.$ Denominator of +ve term is a^2 $a>b$ or $a<b$

$b^2=a^2(-1+e^2)=a^2(e^2-1)$

<u>ATQ</u>: <u>Distance between</u> foci

$2ae=\sqrt{(1-1)^2+(14+12)^2}=26$

$\therefore$ $\boxed{ae=13}$

Mid point of f_1 & f_2 is centre $(h,k)=\left(\frac{1+1}{2},\frac{14-12}{2}\right)$ $=(1,1)$

<u>Eqn. of hyperbola</u> $\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$

$\Rightarrow \frac{(y-1)^2}{a^2}-\frac{(x-1)^2}{b^2}=1$ —— (1)

continued on next page,

Continued for Q No 1:

As $ae = 13$

& $b^2 = a^2(e^2-1)$

$= a^2e^2 - a^2$

$b^2 = 169 - a^2$. sub in eqn (1)

$$\Rightarrow \frac{(y-1)^2}{a^2} - \frac{(x-1)^2}{169-a^2} = 1$$

ATQ Hyperbola passes through (1,6)

∴ will satisfy eqn of Hyperbola.

$$\frac{(6-1)^2}{a^2} - \frac{(1-1)^2}{169-a^2} = 1$$

$$\frac{25}{a^2} - 0 = 1$$

$$\boxed{a^2 = 25}$$

$$\therefore b^2 = 169 - 25 = \boxed{144}$$

Length of L.R : $= 2b^2/a = \frac{2 \times 144}{5} = \boxed{\frac{288}{5}}$

Correct ans. is (3)

Note (i) As in this question, abscissa is same of f_1 & f_2 ∴ nature of Hyperbola is vertical i.e $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ Here a may be > b or a < b.

(ii) L.R is line passing through focus & perpendicular to principal axis. (i.e on which foci lies).

Q.2 Let $f: R \rightarrow R$ be a twice differentiable function such that $f(x+y) = f(x)\,f(y)$ for all $x, y \in R$. If $f'(0) = 4a$ and f satisfy $f''(x) - 3a\,f'(x) - f(x) = 0$, a > 0, then the area of the region

$R = \{(x, y) \mid 0 \le y \le f(ax), 0 \le x \le 2\}$ is:

(1) $e^2 - 1$ (2) $e^4 + 1$

(3) $e^4 - 1$ (4) $e^2 + 1$

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CONCEPT APPLICABLE AND SOLUTION:

ATQ As $f(x+y) = f(x)\cdot f(y)$

In this condition we can assume $f(x)$ as

$f(x) = e^{\lambda x}$

$\therefore f'(x) = \lambda e^{\lambda x} \Rightarrow f'(0) = \lambda e^0 = \lambda = 4a$

$\therefore \boxed{\lambda = 4a}$

$\therefore f(x) = e^{4ax}$

Given eqn is:- $f''(x) - 3a f'(x) - f(x) = 0$

$\frac{d}{dx}(\lambda e^{\lambda x}) - 3a\lambda e^{\lambda x} - e^{\lambda x} = 0$

As $\lambda = 4a$.

$\lambda^2 e^{\lambda x} - 3a\lambda e^{\lambda x} - e^{\lambda x} = 0$

substitute.

$e^{\lambda x}(16a^2 - 12a^2 - 1) = 0$

Cannot be zero

$\therefore a^2 = \frac{1}{4} \Rightarrow a = \frac{1}{2}, a = -\frac{1}{2}$ rejected

$f(x) = e^{4ax} = e^{4x\frac{1}{2}} = e^{2x}$ as $a > 0$

Continued on Next sheet.

FFP QNO 2

$\therefore f(ax) = e^{2ax} = e^{2\times\frac{1}{2}x} = e^{x}$

$\therefore$ Area of region $= \int_0^2 e^x dx$

$= \left[e^x\right]_0^2$

$= \boxed{e^2 - 1}$

Option 1 is correct.

Q.3 Let the triangle PQR be the image of the triangle with vertices (1, 3), (3, 1) and (2, 4) in the line $x + 2y = 2$. If the centroid of ΔPQR is the point (α, β), then $15(\alpha - \beta)$ is equal to:

(1) 24 (2) 19

(3) 21 (4) 22

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CONCEPT APPLICABLE AND SOLUTION:

Let A : (1, 3), B : (3, 1) & C : (2, 4)

centroid of Δ ABC $= \left(\frac{1+3+2}{3}, \frac{3+1+4}{3}\right)$

$= (2, 8/3)$

Image of this point (2, 8/3) will be (α, β) in line $x + 2y = 2$

Image of a pt. in line $ax + by + c = 0$ is given by $\frac{x - x_1}{a} = \frac{y - y_1}{b} = -2\left(\frac{ax_1 + by_1 + c}{(a^2 + b^2)}\right)$

Here: $(x, y) \equiv (\alpha, \beta)$

& $(x_1, y_1) \equiv (2, 8/3)$ Sub in

$$\frac{\alpha - 2}{1} = \frac{\beta - 8/3}{2} = \frac{1 \times 2 + 2 \times \frac{8}{3} - 2}{1^2 + (2)^2}$$

$$= -\frac{32}{15}$$

$\therefore \alpha - 2 = -\frac{32}{15} \Rightarrow \alpha = -\frac{32}{15} + 2 = -\frac{2}{15}$

Continued on next sheet

FPP: Q No 3.

$$\alpha = -\frac{2}{15}$$

$$\frac{\beta - \frac{8}{3}}{2} = -\frac{32}{15}$$

$$\beta - \frac{8}{3} = -\frac{64}{15}$$

$$\beta = -\frac{64}{15} + \frac{8}{3} = \frac{-64+40}{15}$$

$$= -\frac{24}{15}$$

$\therefore$ Value of $15(\alpha - \beta)$

$$= 15\left(-\frac{2}{15} + \frac{24}{15}\right) = \cancel{15}\left(\frac{22}{\cancel{15}}\right)$$

$$= \boxed{22}$$ Ans

Correct option is (4).

Q.4 Let z_1 , z_2 and z_3 be three complex numbers on the circle $|z| = I$ with $\arg(z_1) = \frac{-\pi}{4}$, $\arg(z_2) = 0$ and $\arg(z_3) = \frac{\pi}{4}$.

If $|z_1\bar{z}_2 + z_2\bar{z}_3 + z_3\bar{z}_1|^2 = \alpha + \beta\sqrt{2}, \alpha, \beta \in Z$, then the value of $a^2 + \beta^2$ is:

(1) 24 (2) 41

(3) 31 (4) 29

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CONCEPT APPLICABLE AND SOLUTION:

Different forms of complex number

$$z = \underbrace{x+iy}_{\text{Cartesian form}} = \underbrace{r[\cos\theta + i\sin\theta]}_{\text{Polar form}} = \underbrace{re^{i\theta}}_{\text{Euler form}} = \text{Pt on Argand Diagram } (x, y)$$

$r = |z| = \sqrt{x^2+y^2}$

$\theta = \tan^{-1}\frac{\text{Im } z}{\text{Re } z}$

$|z| = 1$: is circle $x^2+y^2=1$ with centre $(0,0)$ & radius 1

$\therefore z_1 = |z_1| e^{i\theta} = 1 e^{-i\pi/4} = \cos(-\pi/4) + i(-\sin\pi/4)$

$= \cos\pi/4 - i\sin\pi/4$

$z_1 = \left(\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right)$

$$\left\{ \begin{array}{l} \text{As } \cos(-\theta) = \cos\theta \\ \sin(-\theta) = -\sin\theta \end{array} \right.$$

Continued on next page

PPP: Q No 4

$$z_2 = |z_2| e^{i0} = 1.(\cos 0^\circ + i \sin 0^\circ)$$
$$= 1.$$

$$z_3 = |z|_3 e^{i\pi/4} = (1)\left(\cos \pi/4 + i \sin \pi/4\right)$$
$$= \left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)$$

ATQ

$$\left| z_1\bar{z}_2 + z_2\bar{z}_3 + z_3\bar{z}_1 \right| = \alpha + \beta\sqrt{2}$$

$$\left| \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right)1 + (1)\left(\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)\right|^2 = \alpha + \sqrt{2}\beta$$

$$\Rightarrow \left| \left(\frac{1-i}{\sqrt{2}}\right) + \left(\frac{1-i}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)^2 \right|^2 = \alpha + \sqrt{2}\beta$$

$$\Rightarrow \left| \frac{1-i}{\sqrt{2}} + \frac{1-i}{\sqrt{2}} + \cancel{\frac{1}{2}} - \cancel{\frac{1}{2}} + i \right|^2 = \alpha + \beta\sqrt{2}$$

$$\Rightarrow \left| \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - \sqrt{2}i + i \right|^2 = \alpha + \beta\sqrt{2}$$

$$\Rightarrow |\sqrt{2} + (1-\sqrt{2})i|^2$$

$$\Rightarrow 2 + 1 + 2 - 2\sqrt{2} = 5 - 2\sqrt{2} = \alpha + \beta\sqrt{2}$$

compare

$$\alpha = 5 \quad \beta = -2$$

$$\therefore \alpha^2 + \beta^2 = 25 + 4 = \boxed{29} \quad \text{Ans}$$

option (4) is correct

Q.5 **Using the principal values of the inverse trigonometric functions the sum of the maximum and the minimum values of $16((\sec^{-1}x)^2 + (\text{cosec}^{-1} x)^2)$ is:**

(1) $24\pi^2$ **(2) $18\pi^2$**

(3) $31\pi^2$ **(4) $22\pi^2$**

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<u>CONCEPT APPLICABLE AND SOLUTION:</u>

<u>ATQ</u>: $16[(\sec^{-1}x)^2 + (\text{cosec}^{-1}x)^2]$

making perfect square

$= 16[(\sec^{-1}x + \text{cosec}^{-1}x)^2 - 2\sec^{-1}x \cdot \text{cosec}^{-1}x]$

$= 16[(\frac{\pi}{2})^2 - 2(\frac{\pi}{2} - \text{cosec}^{-1}x) \cdot \text{cosec}^{-1}x]$

$= 16[\frac{\pi^2}{4} - \pi\,\text{cosec}^{-1}x + 2(\text{cosec}^{-1}x)^2]$

Taking '2' common.

$= 32[(\text{cosec}^{-1}x)^2 - \frac{\pi}{2}\text{cosec}^{-1}x + \frac{\pi^2}{8}]$

Making again perfect squares

$= 32[(\text{cosec}^{-1}x)^2 - \frac{\pi}{2}\text{cosec}^{-1}x + \frac{\pi^2}{16} - \frac{\pi^2}{16} + \frac{\pi^2}{8}]$

$= 32[(\text{cosec}^{-1}x - \frac{\pi}{4})^2 + \frac{\pi^2}{16}]$

Continued on next sheet.

FPP: Q NO5 :

$$= 32\left[\left(\operatorname{cosec}^{-1}x - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{16}\right] \quad \text{---(1)}$$

For max^{m} value: $\operatorname{cosec}^{-1}x = -\pi/2$

from (1)

Max^{m} value $= 32\left[\left(-\frac{\pi}{2} - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{16}\right]$

$$= 32\left[\frac{9\pi^2}{16} + \frac{\pi^2}{16} = \frac{10\pi^2}{16}\right]$$

$$= \cancel{32}^{2}\left[\frac{10\pi^2}{\cancel{16}}\right] = \boxed{20\pi^2}$$

For Min^{m}: $x = \sqrt{2}$ i.e. $\operatorname{cosec}^{-1}x = \frac{\pi}{4}$

from (1)

$$= 32\left[\left(\frac{\pi}{4} - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{16}\right]$$

$$= \cancel{32}^{2} \times \frac{\pi^2}{\cancel{16}} = \boxed{2\pi^2}$$

$\therefore$ Sum of max^{m} + min^{m} value of (1)

$$= 20\pi^2 + 2\pi^2 = \boxed{22\pi^2} \text{ Ans}$$

option (4) is correct

Q.6 A coin is tossed three times. Let X denote the number of times a tail follows a head. If μ and σ^2 denote the mean and variance of X, then the value of $64(\mu + a^2)$ is:

(1) 51 (2) 48

(3) 32 (4) 64

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CONCEPT APPLICABLE AND SOLUTION:

Sample space when 3 coins are ~~tossed~~ tossed.

$S = \{HHH\ HTH\ HHT\ HTT\ TTT\ TTH\ THT\ THH\}$ $n(S) = 8$.

- Sample space for outcomes when tail does not follow head (once)
  $\{HHH, THH, TTT, TTH\}$
- Sample space when tail follows a head.
  $\{HHT, HTT, HTH, THT\}$

$\therefore$ Probability distribution

$x_i =$	0	1
$p_i =$	$4/8 = 1/2$	$4/8 = 1/2$
$x_i^2 =$	0	1
$p_i x_i^2 =$	$1/2$	$1/2$

Contd. on next sheet

FPP Q.NO 6 :

$$\text{Mean} = \mu = \Sigma p_i x_i = 1/2$$

$$\text{Variance} = (\sigma^2) = \Sigma p_i x_i^2 - (\text{Mean})^2$$

$$= \frac{1}{2} - \left(\frac{1}{2}\right)^2 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

$\therefore$ Value of $64(\mu + \sigma^2)$

$$64\left(\frac{1}{2} + \frac{1}{4}\right) = 64\left(\frac{3}{4}\right) = \boxed{48}$$

option (2) is correct.

Sequence + Series

Q.7 Let a_1, a_2, a_3 .... be a G.P. of increasing positive terms, if $a_1 a_5 = 28$ and $a_2 + a_4 = 29$, the a_6 is equal to

(1) 628 (2) 526

(3) 784 (4) 812

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CONCEPT APPLICABLE AND SOLUTION:

G.P: increasing, +ve terms.

In G.P $\Rightarrow$ $t_n = ar^{n-1}$ a: first term, r: common ratio

ATQ

$a_1 a_5 = 28$

$(a)(ar^4) = 28 \Rightarrow a^2 r^4 = 28$. — (1)

$a_2 + a_4 = 29$

$ar + ar^3 = 29 \Rightarrow ar(1+r^2) = 29$ — (2)

eliminating a from (2) using (1)

from (1) $\Rightarrow$ $ar = (ar)^2 \cdot r^2 = 28$

from (2) $\Rightarrow$ $ar = \frac{29}{1+r^2}$ — (A)

$\therefore$ $\left(\frac{29}{1+r^2}\right)^2 \cdot r^2 = 28$.

$\Rightarrow$ $841 r^2 = 28(r^4 + 2r^2 + 1)$

$841 r^2 = 28r^4 + 56r^2 + 28$

$28r^4 - 785r^2 + 28 = 0$

$\therefore$

contd on next sheet

Eqn. ⑦:

$$(r^2 - 28)(28r^2 - 1) = 0$$

$$r^2 = 28 \quad \text{or} \quad r^2 = 1/28$$

As G.P is inc $\therefore r^2 = 28$

$$\therefore a_6 = ar^5 = a(\cancel{\sqrt{28}})^5$$

To find a_6 using 'a':

$$= \underline{ar}\cdot r^4 \text{ from (A)}$$

$$\left(\frac{29}{1+r^2}\right) r^4$$

$$= \left(\frac{29}{1+28}\right)(\sqrt{28})^4$$

$$= \boxed{784}$$

correct option is (3)

Q.8 Let $L_1: \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $L_2: \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ be two lines. Then which of the following points lies on the line of the shortest distance between L_1 and L_2?

(1) $\left(-\frac{5}{3}, -7, 1\right)$ (2) $\left(2, 3, \frac{1}{3}\right)$

(3) $\left(\frac{8}{3}, -1, \frac{1}{3}\right)$ (4) $\left(\frac{14}{3}, -3, \frac{22}{3}\right)$

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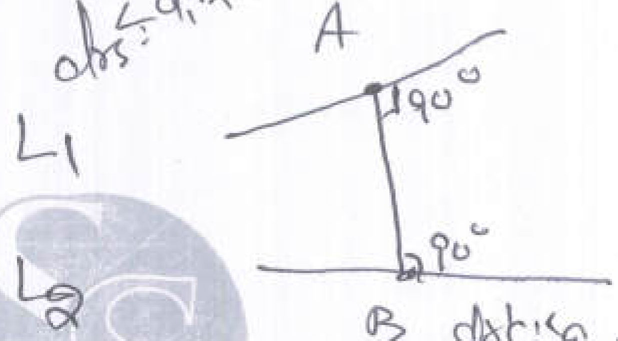
<u>CONCEPT APPLICABLE AND SOLUTION:</u>

drs: $\langle a_1, b_1, c_1 \rangle : \langle 2, 3, 4 \rangle$



B drs: $\langle a_2, b_2, c_2 \rangle : \langle 3, 4, 5 \rangle$

$L_1: \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$ say

$\underbrace{x = 2\lambda+1,\ y = 3\lambda+2,\ z = 4\lambda+3}_{}$ Let this's general pt be A (set 1)

$L_2: \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5} = \mu$ (say)

$\underbrace{x = 3\mu+2,\ y = 4\mu+4}_{}$ $z = 5\mu+5$ (set 2)

Let this's general pt on line be B.

d.r.s of line AB:
$a = x_2 - x_1 = 2\lambda - 3\mu - 1$
$b = y_2 - y_1 = 3\lambda - 4\mu - 2$
$c = z_2 - z_1 = 4\lambda - 5\mu - 2$

fpp Q No 8

As L_1 ⊥ to AB : $aa_1+bb_1+cc_1=0$

L_2 ⊥ to AB : $aa_2+bb_2+cc_2=0$

Solve these two equations in λ & μ. & simplification

$\lambda = 1/3$ & $\mu = -1/6$

Sub in set 1   Sub in set 2

Co-ordinate of Pt A : $\left(\frac{5}{3}, 3, \frac{13}{3}\right) = (x_1, y_1, z_1)$

Co-ordinate of Pt B : $\left(\frac{3}{2}, \frac{10}{3}, \frac{29}{6}\right) = (x_2, y_2, z_2)$

∴ By two point form : eq of line AB

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

$$\frac{x-5/3}{3/2-5/3} = \frac{y-3}{10/3-3} = \frac{z-13/3}{29/6-13/3}$$

Eqn of line AB

$$\Rightarrow \boxed{x-5/3 = \frac{y-3}{-2} = \frac{z-\frac{13}{3}}{3}}$$

$\text{Pt}\left(\frac{14}{3}, -3, \frac{22}{3}\right)$ lie on line AB

option (4) is correct

Note : for shortest : s.d line will be ⊥ to both the given lines

Q.9 **The product of all solutions of the equation** $e^{5(\log_e x)^2+3} = x^8, x > 0$, **is:**

(1) $e^{8/5}$ **(2)** $e^{6/5}$

(3) e^{2} **(4)** e

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CONCEPT APPLICABLE AND SOLUTION:

Given eqn is:

$$e^{5(\log_e x)^2+3} = x^8, \quad x > 0$$

Taking $\log_e$ of both sides

$$5(\log_e x)^2 + 3 = 8\log_e x.$$

$$5(\log_e x)^2 - 8\log_e x + 3 = 0$$

Let $\log_e x = y$

$$5y^2 - 8y + 3 = 0 \quad \text{—— (A)}$$

$$5y^2 - 5y - 3y + 3 = 0$$

$$5y(y-1) - 3(y-1) = 0$$

$$(y-1)(5y-3) = 0 \Rightarrow y = 1 \text{ or } y = 3/5$$

For (A) Sum of roots: $(\alpha+\beta = -b/a)$

$$y_1 + y_2 = \frac{8}{5}$$

B.S

$$\log_e x_1 + \log_e x_2 = 8/5$$

$$\log_e x_1 x_2 = 8/5$$

$\therefore$ Product of roots: $x_1 x_2 = e^{8/5}$

Option (1) is correct

Q.10 If $\sum_{r=1}^{n} T_r = \frac{(2n-1)(2n+1)(2n+3)(2n+5)}{64}$, then $\lim_{n\to\infty} \sum_{r=1}^{n} \left(\frac{1}{T_r}\right)$ is equal to:

(1) 1 (2) 0

(3) $\frac{2}{3}$ (4) $\frac{1}{3}$

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CONCEPT APPLICABLE AND SOLUTION:

We know that nth term is given by also

$t_n = S_n - S_{n-1}$

$S_n = \frac{(2n-1)(2n+1)(2n+3)(2n+5)}{64}$ — (1)

replace n by n-1 in eq (1)

$S_{n-1} = \frac{(2n-3)(2n-1)(2n+1)(2n+3)}{64}$ — (2)

$\therefore t_n = S_n - S_{n-1}$

$= \frac{(2n-1)(2n+1)(2n+3)}{64}\left[2n+5-(2n-3)\right]$

$\boxed{t_n = \frac{1}{8}(2n-1)(2n+1)(2n+3)}$

$= \left[\frac{1}{(2n-1)(2n+1)} - \frac{1}{(2n+1)(2n+3)}\right]$

Contd Next Page

APP 010

$$\left[\frac{1}{(2n-1)(2n+1)} - \frac{1}{(2n+1)(2n+3)}\right]$$

$$\left(\frac{2n+3 - 2n+1}{(2n-1)(2n+1)(2n+3)}\right) = \frac{4}{(2n-1)(2n+1)(2n+3)}$$

$$\therefore \sum_{n=1}^{n} \frac{1}{T_n} = \frac{8}{4}\left[\frac{1}{(2n-1)(2n+1)} - \frac{1}{(2n+1)(2n+3)}\right]$$

Telescopic series

$$\lim_{n\to\infty} \sum_{n=1}^{n} \frac{1}{T_n} = 2\lim_{n\to\infty}\left[\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{3.5} - \frac{1}{5.7} \text{------} 0\right]$$

$$= \boxed{\frac{2}{3}}$$ Ans

option (3) is correct

Q.11 **From all the English alphabets, five letters are chosen and are arranged in alphabetical order. The total number of ways, in which the middle letter is 'M' is:**

(1) 14950	**(2) 6084**
(3) 4356	**(4) 5148**

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CONCEPT APPLICABLE AND SOLUTION:

1 2 3 4 5

12 ⟵ M ⟶ 13 alphabets.

a b c d e f g h i j k l m

12 alphabets

Out of 12 : 2 are to be selected

$^{12}C_2$

Out of 13 : 2 are to be selected

By prep multiplication:

$\therefore$ Total No of ways $= {}^{12}C_2 \cdot {}^{13}C_2$

$= \left(\frac{12!}{10! \times 2!}\right)\left(\frac{13!}{11! \times 2!}\right)$

$= \left(\frac{12 \times 11}{2}\right)\left(\frac{13 \times 12}{2}\right) = 66 \times 78$

$= \boxed{5148}$

option (4) is correct.

Q.12 Let $x = x(y)$ be the solution of the differential equation $y^2dx + \left(x - \frac{1}{y}\right) dy = 0$. If $x(I) = I$ then $x\left(\frac{1}{2}\right)$ is:

(1) $\frac{1}{2} + e$

(2) $\frac{1}{2} + e$

(3) $3 - e$

(4) $3 + e$

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CONCEPT APPLICABLE AND SOLUTION:

DE : $y^2 dx + (x - \frac{1}{y}) dy = 0$ — (1)

Given. $x(1) = 1$ i.e when $x = 1, y = 1$.

To find x_1 when $y = \frac{1}{2}$.

Dividing eq (1) by dy

$$y^2 \frac{dx}{dy} + x - \frac{1}{y} = 0$$

$$\frac{dx}{dy} + \left(\frac{1}{y^2}\right) x = \frac{1}{y^3}$$

Type: FOL eqn of type $\frac{dx}{dy} + Rx = S$

$$IF = e^{\int \frac{1}{y^2} dy} = e^{-1/y}$$

$\therefore$ soln is given by

$$x \cdot IF = \int S \cdot IF \cdot dy.$$

$$x \cdot e^{-1/y} = \int \frac{1}{y^3} \cdot e^{-1/y} \, dy \quad \text{— (A)}$$

Contd on next page.

FPP Q12

Let $\frac{1}{y} = t \Rightarrow -\frac{1}{y^2}dy = dt$

In (A) RHS

$$= -\int \frac{1}{y} e^{-1/y} \cdot \left(-\frac{1}{y^2}dy\right)$$

$$= -\int \underset{\text{I}}{t}\,\underset{\text{II}}{e^{-t}}\, dt$$

By parts ILATE
I II

$$\text{I} \cdot \int \text{II} - \int \left[\frac{d}{dt}\text{I} \cdot \int \text{II}\right] dt$$

$$= -\left[(t)\, e^{-t}(-1) - \int (1)\, e^{-t}(-1)\, dt \right]$$

$$= -\left[-te^{-t} + \int e^{-t}\, dt \right]$$

$$= -\left[-te^{-t} - e^{-t} \right] + C$$

$$= te^{-t} + e^{-t} + C$$

Backsub

In (A)

$$\therefore \boxed{x e^{-1/y} = \frac{1}{y} e^{-1/y} + e^{-1/y} + C}$$

To find 'c'

At $x=1, y=1 \Rightarrow e^{-1} = e^{-1} + e^{-1} + c \Rightarrow c = -1/e$

$$x e^{-1/y} = \frac{1}{y} e^{-1/y} + e^{-1/y} + (-1/e)$$

To find x, at $y = 1/2 \Rightarrow x e^{-2} = 2e^{-2} + e^{-2} - e^{-1}$

$\therefore x = 2 + 1 - e = \boxed{3-e}$ Ans

option (3) is correct

Q.13 **Let the parabola $y = x^2 + px - 3$, meet the coordinate axes at the points P, Q and R. If the circle C with centre at (−1, −1) passes through the points P, Q and R, then the area of ΔPQR is:**

(1) 4 **(2) 6**

(3) 7 **(4) 5**

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CONCEPT APPLICABLE AND SOLUTION:

Eqn. of Parabola: $y = x^2 + px - 3$

ATQ Parabola meets co-ordinates axis at P, Q & R

Let Pt P be $(\alpha, 0)$, Q be $(\beta, 0)$ & R be $(0, -3)$
By subs $x = 0$

Eqn of circle in central form
$(x-h)^2 + (y-k)^2 = r^2$
centre : (h, k) radius $= r$

Here $(h, k) = (-1, -1)$
∴ Eqn of circle : $(x+1)^2 + (y+1)^2 = r^2$
ATQ it also passes th. R ie $(0, -3)$
$1 + 4 = r^2 \Rightarrow r^2 = 5$

∴ Eqn of circle: $(x+1)^2 + (y+1)^2 = 5$ — (A)
This circle also passes through the pt $(\alpha, 0)$ & $(\beta, 0)$ so will satisfy eqn (A)

Next Page.

PPP, Q No 13

$(\alpha+1)^2+(0+1)^2=5$

$(\alpha+1)^2=5-1=4$

$\alpha+1=\pm 2$

$\alpha=\pm 2-1=\underbrace{1 \text{ or } -3}$

∴ Pts P, Q, R are respectively

P : (1, 0)

Q : (−3, 0)

R : (0, −3)

∴ Area of Δ PQR

$$=\frac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ -3 & 0 & 1 \\ 0 & -3 & 1 \end{vmatrix}$$

expand. along R1

$$=\frac{1}{2}\left[1(0+3)-0+1(9-0)\right]$$

$$=\frac{1}{2}[3+9]=\frac{12}{2}=\boxed{6 \text{ sq. units}}$$

option (2) is correct

Q.14 A circle C of radius 2 lies in the second quadrant and touches both the coordinate axes. Let r be the radius of a circle that has centre at the point (2, 5) and intersects the circle C at exactly two points. If the set of all possible values of r is the interval (α, β), then $3\beta - 2\alpha$ is equal to:

(1) 15 **(2) 14**

(3) 12 **(4) 10**

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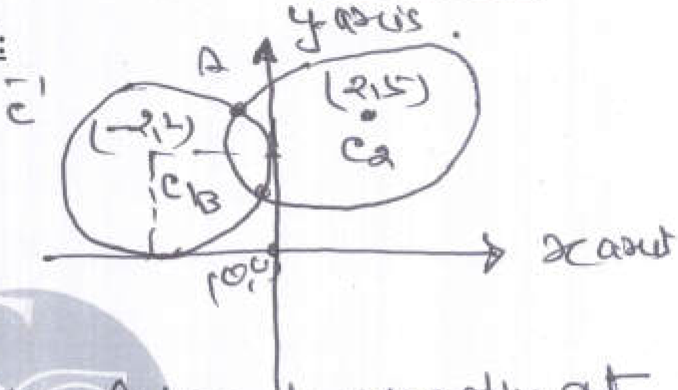
CONCEPT APPLICABLE AND SOLUTION:



C_1: C: centre $(-2,2)$
r_1 = radius 2
C_2: C: centre $(2,5)$
r_2 = radius r

Let a. ATQ both circles intersects exactly at two points. Let pts be A & B.

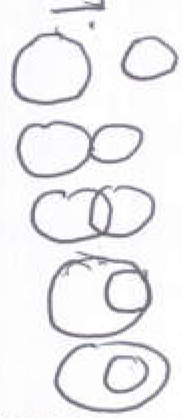
C_1C_2 = Distance between the centre C_1 & C_2

$= \sqrt{(2+2)^2+(5-2)^2} = \sqrt{16+9} = 5$ units

& $r_1+r_2 = |2+r|$

Additional Ary.

	condition	Fig.	no. of common tangents
(i)	$\lvert C_1C_2\rvert > r_1+r_2$		4
(ii)	$= r_1+r_2$		3
(iii)	$(r_1-r_2) < C_1C_2 < r_1+r_2$		2.
(iv)	$C_1C_2 = \lvert r_1-r_2\rvert$		1
(v)	$C_1C_2 < \lvert r_1-r_2\rvert$		0



RFP : QNo14

In this question, condition applicable is $\underbrace{|r_1 - r_2| < \underbrace{C_1C_2 < r_1 + r_2}}_{\text{intersect exactly at two points.}}$

$$|r_1 - r_2| < C_1C_2 < r_1 + r_2$$

$$|r - 2| < 5 < r + 2$$

$5 < r + 2$

$\boxed{r > 3}$

$|r-2| < 5$

+ve : $r - 2 < 5$

$\boxed{r < 7}$

−ve : $-r + 2 < 5$

$r > -3$ } rejected

i.e. $\boxed{3 < r < 7}$

$\therefore \ (\alpha, \beta) = (3, 7)$

$\Rightarrow$ Value of $(3\beta - 2\alpha)$

$= 3 \times 7 - 2 \times 3$

$= 21 - 6 = \boxed{15}$ Ans

option (1) is correct

Q.15 Let for $f(x) = 7\tan^8 x + 7\tan^6 x - 3\tan^4 x - 3\tan^2 x$ $I_1 = \int_0^{\pi/4} f(x)\,dx$ and $I_2 = \int_0^{\pi/4} x f(x)\,dx$. Then $7I_1 + 12I_2$ is equal to

(1) 2π (2) π

(3) 1 (4) 2

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CONCEPT APPLICABLE AND SOLUTION:

$f(x) = 7\tan^6 x(\tan^2 x + 1) - 3\tan^2 x(\sec^2 x - 1)$

$= (7\tan^6 x - 3)\cdot(1 + \tan^2 x)$

$= (7\tan^6 x - 3\tan^2 x)\sec^2 x$

$= 7\tan^6 x\sec^2 x - 3\sec^2 x$

$\therefore I_1 = 7\int_0^{\pi/4} \tan^6 x\sec^2 x\,dx - 3\int_0^{\pi/4} \tan^2 x\sec^2 x\,dx$

Put $\tan x = t$

$\sec^2 x\,dx = dt$

changing limit at $x = 0,\ t = 0$

at $x = \pi/4,\ t = \tan\pi/4 = 1$

$\therefore I_1 = 7\left[\frac{t^7}{7}\right]_0^1 - 3\left[\frac{t^3}{3}\right]_0^1$

$= (t^7)_0^1 - (t^3)_0^1 = 1 - 1 = 0$

$\boxed{\therefore I_1 = 0}$

next page

FPP QM015

$$I_9 = \int_0^{\pi/4} x\left(7\tan^6 x - 3\tan^2 x\right).\sec^2 x$$

Put $\tan x = t$

$\sec^2 x\,dx = dt$

change limit: $x = 0,\ t = 0$

$x = 1,\ t = 1$

$$= \int \underset{I}{\tan^{-1} t}\left(\underset{II}{7t^6 - 3t^2}\right) dt$$

ILATE

I II

By parts

$$I.\int II\,dt - \int\left[\frac{d}{dt}(I)\int II\,dt\right]dt$$

$$\therefore I_9 = \tan^{-1}t.(t^7 - t^3) - \int \frac{1}{1+t^2}.(t^7 - t^3)$$

$$= \left[\tan^{-1}t.(t^7 - t^3)\right]_0^1 - \int_0^1 \frac{t^3(t^4-1)}{(1+t^2)}\,dt$$

$$= 0 - \int_0^1 t^3(1+t^2)\,dt$$

$$- \int_0^1 (-t^3 + t^5)\,dt$$

$$0 - \left[-\frac{t^4}{4} + \frac{t^6}{6}\right]_0^1$$

$$0 + \left[-\frac{1}{6} + \frac{1}{4}\right] = \frac{-2+3}{12} = \frac{1}{12}$$

$$\boxed{I_9 = 1/12}$$

$\therefore$ Value of $7I_7 + 12I_9 = 7\times 0 + \cancel{12}\times\frac{1}{\cancel{12}}$

$= \boxed{1}$

Option (3) is correct.

Q.16 Let $f(x)$ be a real differentialble function such that $f(0) = I$ and $f(x+y) = f(x)f'(y) + f'(x)\,f(y)$ for all $x, y \in R$. Then

$$\sum_{n=1}^{100} \log_e f(n) \text{ is equal to:}$$

(1) 2384 (2) 2525

(c) 5220 (4) 2406

(JEE-Main 2025)

CONCEPT APPLICABLE AND SOLUTION:

ATQ $f(x+y) = f(x)f'(y) + f'(x)f(y)$

Let $x=0, y=0$

$f(0) = f(0)f'(0) + f'(0)f(0)$

As $f(0) = 1$

$1 = 1f'(0) + f(0).1$

$1 = 2f'(0) \Rightarrow \boxed{f'(0) = 1/2}$

When $y = 0$

$f(x) = f(x)f'(0) + f'(x)f(0)$

$= f(x)\frac{1}{2} + f'(x) \times 1$

$\therefore f'(x) = \frac{1}{2}f(x) \Rightarrow$ Diff of fn is fn it self a integrate exp function.

$\therefore f(x) = e^{x/2} + c$

$f(0) = e^0 + c \Rightarrow \boxed{c = 0}$

$\therefore f(x) = e^{x/2}$

$\log_e f(x) = \log_e e^{x/2} = x/2$

Next page

FPP. Q.No16

$$\therefore \sum_{n=1}^{100} \log_e f(n) = \sum_{n=1}^{100} \frac{n}{2}$$

$S_1 = 1+2+3+ \text{———} + n =$ sum of n natural nos $= \frac{n(n+1)}{2}$

$$= \frac{1}{2} \frac{(n)(n+1)}{2}$$

Sub $n = 100$

$$= \frac{1}{2} \times \frac{100 \times 101}{2}$$

$$25 \times 101 = \boxed{2525}$$

option (2) is correct.

Q.17 Let $A = \{1, 2, 3, \ldots\ldots..10\}$ and $B = \{\frac{m}{n} = m, n \in A, m < n,\ ged\,(m, n) = 1\}$

Then n(B) is equal to:

(1) 31 **(2) 36**

(3) 37 **(4) 29**

(JEE-Main 2025)

CONCEPT APPLICABLE AND SOLUTION:

ATQ $A = \{1, 2, 3 - - - 10\}$

$B = \{\frac{m}{n} = m, n \in A\ \ m < n, \gcd(m,n) = 1\}$

For $m = 1$ $\quad n = 2, 3 - - - 10 \Rightarrow$ 9 cases 9

$= 2$ $\quad n = 3, 5, 7, - -$ (odd nos) 4 cases 13

$= 3$ $\quad n = 4, 5, 7, 8, 10$ 5 cases 18

$= 4$ $\quad n = 5, 7, 9,$ 3 cases 21

$= 5$ $\quad n = 6, 7, 8, 9.$ 4 cases 25

$= 6$ $\quad n = 7$ 1 cases 26

$= 7$ $\quad n = 8, 9, 10$ 3 cases 29

$= 8$ $\quad n = 9$ 1 case 30

$= 9$ $\quad n = 10$ 1 case 31

$\boxed{\therefore n(B) = 31}$

option 1 is correct

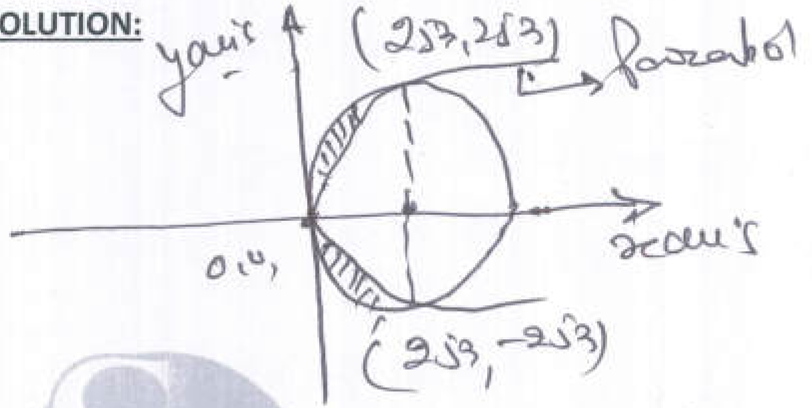
Q.18 The area of region, inside the circle $(x-2\sqrt{3})^2+y^2=12$ and outside the parabola $y^2=2\sqrt{3}\,x$ is

(1) $6\pi-8$ (2) $3\pi-8$

(3) $6\pi-16$ (4) $3\pi+8$

(JEE-Main 2025)

CONCEPT APPLICABLE AND SOLUTION:



$\underbrace{(x-2\sqrt{3})^2+y^2=12}$ is circle with centre $(2\sqrt{3}, 0)$

radius $=\sqrt{12}$
$=2\sqrt{3}$

$\underbrace{y^2=2\sqrt{3}x}$ is parabola.

Note: R Handed, vertex $(0,0)$

Pt of Intersection of circle & Parabola

$(x-2\sqrt{3})^2+(2\sqrt{3}x)=12$

$x^2+\cancel{12}-4\sqrt{3}x+2\sqrt{3}x=\cancel{12}$

$x^2-2\sqrt{3}x=0$

$x=0$ & $x=2\sqrt{3}$

$\therefore y=0$ & $y^2=(2\sqrt{3})(2\sqrt{3})$

$y=\pm 2\sqrt{3}$

Next Page

FPP. Q.No18:

Area of shaded region is

$= \text{Area of semicircle} - 2\int_0^{2\sqrt{3}} y\,dx$ Parabola.

$\frac{1}{2}\pi \times 12 - 2\int_0^{2\sqrt{3}} (2\sqrt{3})^{1/2} \cdot x^{1/2}\,dx$

$6\pi - 2\times(2\sqrt{3})^{1/2}\left[\frac{x^{3/2}}{3/2}\right]_0^{2\sqrt{3}}$

$= 6\pi - 2(2\sqrt{3})^{1/2} \cdot \frac{2}{3} (2\sqrt{3})(2\sqrt{3})^{1/2}$

$= 6\pi - \frac{4}{3} \times 2\sqrt{3} \times 2\sqrt{3}$

$= 6\pi - \frac{16}{3} \times 3 = \boxed{(6\pi - 16) \text{ sq. units}}$

Ans

Option (3) is correct

Q.19 Two balls are selected at random one by one without replacement from a bag containing 4 white and 6 black balls. If the probability that the first selected ball is black, given that the second selected ball is also black, is $\frac{m}{n}$, where gcd(m, n) = 1, then $m + n$ is equal to :

(1) 14 (2) 4

(3) 11 (4) 13

(JEE-Main 2025)

CONCEPT APPLICABLE AND SOLUTION:

4 white
6 Black

$\rightarrow$ 2 balls are drawn one by one without replacement

Total no of balls = 6+4 = 10

Let p_1 be the probability first ball is black

p_2 be the prob second ball is black

To find

$P(p_1|p_2)$ events

$$\frac{\left(\frac{6}{10}\right)\left(\frac{5}{9}\right)}{\left(\frac{6}{10}\cdot\frac{5}{9}\right)+\left(\frac{6}{10}\right)\left(\frac{4}{9}\right)}$$

By Baye's theorem

$$\frac{5}{5+4} = \frac{5}{9}$$ gcd = 1

$\therefore m+n = 5+9 = \boxed{14}$ Ans

Q.20 The number of non-empty equivalence relations on the set {1,2,3} is:

(1) 6 **(2) 7**

(3) 5 **(4) 4**

(JEE-Main 2025)

CONCEPT APPLICABLE AND SOLUTION:

Short cut Method: To find no of non empty equivalence relations

No of elements in the set	
1	1
2	1 2
3	2 3 5
4	5 7 10 15
5	15 20 27 37 52

No of non empty equivalence relation when $n=3$

non empty no of equivalence relation, no of elements in the set is 5

Procedure: • Add no one below the other to get no on the right.

• Last no of each row gives the first no of the next row

• Largest no in each row gives the largest no of equivalence relations on a set

ATQ: $n=3$, ∴ No of nonempty equivalence relations on the set {1,2,3} is $\boxed{5}$ Ans

Option (3) is correct.

Q.21 Let the function,

$$f(x) = \begin{cases} -3ax^2 - 2, & x < 1 \\ a^2 + bx, & x \geq 1 \end{cases}$$

Be differentiable for all $x \in R$, where $a > 1, b \in R$. If the area of the region enclosed by $y = f(x)$ and the line $y = -20$ is $\alpha + \beta\sqrt{3}, \alpha, \beta, \in Z$, then the value of $\alpha + \beta$ is _______.

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CONCEPT APPLICABLE AND SOLUTION:

- If a fn is differentiable, then it is continuous also
- If fn is continuous, it will be differentiable → it may or may not
- for continuity: LHL = RHL = f(a)
- for Differentiability: LHD = RHD

Differentiable at $x=1$ means given fn is continuous also. As there are two variables a & b applying both conditions to find a and b.

LHL = RHL = f(1) ⇒ $-3a-2 = a^2+b$ — (1)

LHD = RHD ⇒ $-6ax = b$, subs $x = 1$

$\boxed{b = -6a}$ — (2)

solving (1) & (2) $\boxed{a = 2, b = -12}$

FPP. Q21.

Parabola : $y = -6x^2 - 2$ } for $x<1$

$y = 4 - 12x$ } for $x \geq 1$

Line 1 :

Line 2 : $y = -20$

Pt of intersection of P & Line 1

$x = \underbrace{-\sqrt{3}, \sqrt{3}}_{\text{rejected, as it is applicable for } x<1}$

Pt of Intersection of Line 1 & 2

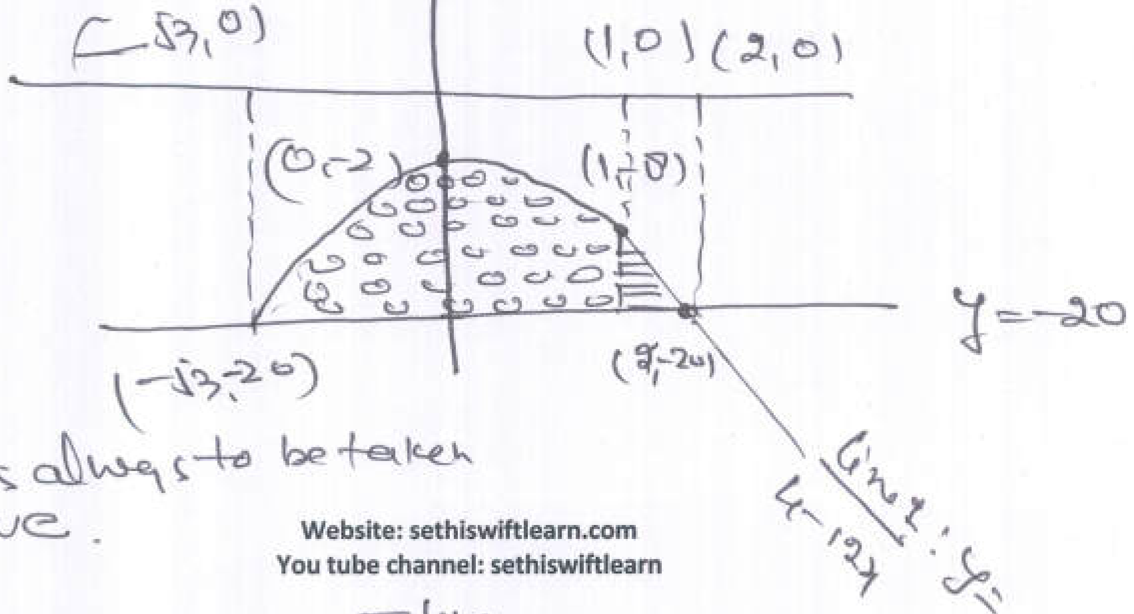
Pt $(1,-8)$

$P \Rightarrow y+2 = -6x^2 \Rightarrow$ Para downward vertex $(0,-2)$

To find reqd Area : ie $\circ\circ$ + $\equiv$

$$\underbrace{\int_{-\sqrt{3}}^{1} y\,dx}_{\text{(Line 2 - Parabola)}} - \underbrace{\int_{1}^{2} y\,dx}_{\text{Line 2 - Line 1}}$$

(A) (B)



• Area is always to be taken as +ve.

FPP Q.21

Area of A :

$$\int_{-\sqrt{3}}^{1} \left[(-20x) - (-6x^2-2)\right] dx$$

$$-\left[20x\right]_{-\sqrt{3}}^{1} - \left[-2x^3-2x\right]_{-\sqrt{3}}^{1}$$

$$= -\left[20+20\sqrt{3}\right] - \left[-2(1+3\sqrt{3}) - 2(1+\sqrt{3})\right]$$

$$= -\underbrace{\left[20+20\sqrt{3}\right]}_{\text{magnitude of this}} - \left[-2-6\sqrt{3}-2-2\sqrt{3}\right]$$

$$\underbrace{\left[-4-8\sqrt{3}\right]}_{\text{magnitude of this}}$$

$$= 20+20\sqrt{3} - (4+8\sqrt{3})$$

$$= (16+12\sqrt{3}) \text{ sq units.}$$

Area of B : $\equiv \int_{1}^{2} \left[-20 - (4-12x)\right]$,

$$= -\left[20x\right]_1^2 - \left[4x-6x^2\right]_1^2$$

$$= -\left[20(2-1)\right] - \left[4(2-1) - 6(4-1)\right]$$

$$-\underbrace{20}_{\text{magnitude write}} - \left[4-18\right]$$

$$- \underbrace{(-14)}_{\text{magnitude w.r.}}$$

$$20 - 14 = 6$$

Cont on Next P

Feb. 2021

$\therefore$ Requisite Area

$= \text{---} + \text{---}$

$= (16 + 12\sqrt{3}) + (6)$

$= 22 + 12\sqrt{3}$

comparing with $\alpha + \beta\sqrt{3}$

$\alpha = 22, \quad \beta = 12$

$\therefore \boxed{\alpha + \beta = 34}$

→ to fill in the blanks

Q.22 If

$$\sum_{r=0}^{5} \frac{{}^{11}C_{2r+1}}{2r+2} = \frac{m}{n}, \gcd(m,n)$$
$$= 1, \gcd(m,n) = 1, \text{then } m-n \text{ is equal to \_\_\_\_\_\_\_\_.}$$

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CONCEPT APPLICABLE AND SOLUTION:

$$(1+x)^n = C_0 + C_1x + C_2x^2 + C_3x^3 + \cdots + C_nx^n \quad \text{—(1)}$$

Integrating both sides w.r.t x with limits 0 to 1

$$\int_0^1 (1+x)^n = \left[C_0x + C_1\frac{x^2}{2} + C_2\frac{x^3}{3} + \cdots + C_n\frac{x^{n+1}}{n+1}\right]$$

Sub here $x = 1$

$$\frac{(1+x)^{n+1}}{n+1} = \left[\quad\right] \quad \text{—(A)}$$

$$\Rightarrow \boxed{\frac{2^{n+1}-1}{n+1} = C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \cdots + \frac{C_n}{n+1}}$$

Integrating (1) w.r.t x in limits -1 to 0

$$\boxed{\frac{+1}{n+1} = C_0 - \frac{C_1}{2} + \frac{C_2}{3} + \cdots (-1)^n\frac{C_n}{n+1}} \quad \text{—(B)}$$

(A) − (B)

$$2\left[\frac{C_1}{2} + \frac{C_3}{4} + \cdots \quad\right] = \frac{2^{n+1}-1}{n+1} - \frac{1}{n+1} \quad \text{—(C)}$$

PPP. QNo22 :

Sub $x = 11$ in eqn (c)

$$2\left[\frac{C_1}{3}+\frac{C_3}{4}+ \text{———} +\frac{C_{11}}{12}\right] = \frac{2^{12}-1}{12}-\frac{1}{12}$$

$$\therefore \left(\frac{C_1}{3}+\frac{C_3}{4}+ \text{———} +\frac{C_{11}}{12}\right) = \frac{1}{2}\left[\frac{2^{12}-2}{12}\right] \quad \text{— (4)}$$

APQ : $\displaystyle\sum_{r=0}^{5} \frac{{}^{11}C_{2r+1}}{2r+2}$

Hay series

$$\frac{C_1}{3}+\frac{C_3}{4}+ \text{——} = \frac{m}{n} \quad \text{— (5)}$$

Comp' eqn (4) & (5)

$$\frac{1}{2}\left[\frac{2^{12}-2}{12}\right] = \frac{m}{n}.$$

$$\frac{2^{11}-2}{12} = \frac{m}{n}$$

$$\Rightarrow \frac{64\times 32-1}{12}$$

$$\Rightarrow \frac{2057}{12} = \frac{m}{n} \quad \text{GCD of } m, n = 1$$

$\therefore$ $m = 2057$, $n = 12$ $\quad \therefore$ $m-n = 2057-12 = \boxed{2035}$

— Tai's fill in the blanks

Q.23 Let A be a square matrix of order 3 such that $\det(A) = -2$ and $\det(3\text{adj}(-6\text{adj}(3A))) = 2^{m+n}.3^{mn}, m > n$. Then $4m + 2n$ is equal to ________.

(JEE-Main 2025)

CONCEPT APPLICABLE AND SOLUTION:

Properties applicable:

$|\lambda A| = \lambda^n . |A|$

$|\text{Adj} A| = |A|^{n-1}$

where n is order of determinant.

$\det(3\text{adj}(-6\text{adj}(3A))) = 2^{m+n} . 3^{mn} \quad m>n$

To find value of $4m+2n$.

$3^3 \det \underbrace{\text{adj}(-6\text{ad}(3A))}_{2}$

$3^3 (-6 \text{Adj} 3A)^2$

$3^3((-6)^3)^2 (\det(\text{Adj} 3A))^2 \rightarrow \underbrace{[(|A|^2)]^{3-1}}_{\text{concept}}$

$3^3 . 6^6 . (\det(3A))^4$

$3^3 . 6^6 . (3^3)^4 |A|^4$

$3^3 . 3^6 . 2^6 . 3^{12} (-2)^4$

$3^3 . 3^6 . 2^6 . 3^{12} . 2^4 = 3^{21} . 2^{10} = 3^{mn} . 2^{m+n} \quad m>n$

$\therefore 3^{mn} . 2^{m+n} = 3^{7.3} . 2^{7+3}$ As $m>n$, $m=7, n=3$

$\therefore$ Value of $4m+2n = 4\times7 + 2\times3 = 28+6 = \boxed{34}$

Ans

Q.24 Let $L_1: \frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0}$ and $L_2: \frac{x-2}{2} = \frac{y}{0} = \frac{z+4}{a}$, $a \in R$, be two lines, which intersect at the point B. If P is the foot of perpendicular form the point $A(1,1,-1)$ on L_2, then the value of $26\,a(PB)^2$ is ________.

(JEE-Main 2025)

<u>CONCEPT APPLICABLE AND SOLUTION:</u>

To find point of intersection of L_1 & L_2:

L_1: $\frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0} = \lambda$ say

$$\left.\begin{aligned} x &= 3\lambda+1 \\ y &= -\lambda+1 \\ z &= -1 \end{aligned}\right\}$$ Let this general pt on line be 'B'

Also L_2: $\frac{x-2}{2} = \frac{y}{0} = \frac{z+4}{a} = \mu$

$$\left.\begin{aligned} x &= 2\mu+2 \\ y &= 0 \\ z &= a\mu-4 \end{aligned}\right\}$$ Let this general pt on line be 'B'

$\therefore$ $3\lambda+1 = 2\mu+2 \Rightarrow 3\lambda - 2\mu = +1$ — (A)

$-\lambda+1 = 0 \Rightarrow \boxed{\lambda = 1}$

sub $\lambda = 1$ in (A)

$\boxed{\mu = 1}$

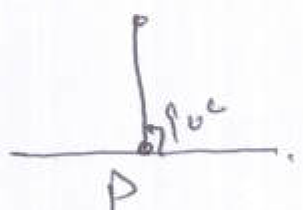
$-1 = a\mu - 4$

$\boxed{a\mu = 3}$

$\boxed{a = 3}$

APPROACH

To find foot of perpendicular on a line:



$L_2: \frac{x-2}{2} = \frac{y}{0} = \frac{z+4}{3} = \beta$ (say) L_2:

dr's of line L_2: 2, 0, 3

i.e $\langle 2, 0, 3 \rangle$
a_1, b_1, c_1

$\langle 2\beta+2, 0, 3\beta-4 \rangle$
Let its general pt on L_2 be P. — (B)

$\therefore$ dr's of AP: $2\beta+2-1 = 2\beta+1$
$0-1 = -1$
$3\lambda-4+1 = 3\beta-3$

$\therefore$ dr's of AP $\langle a_2, b_2, c_2 \rangle = \langle 2\beta+1, -1, 3\beta-3 \rangle$

As AP $\perp$ to L_2: $a_1a_2+b_1b_2+c_1c_2 = 0$

On sub & simplification, we get

$\boxed{\beta = 9/13}$

Sub β in (B) above $\Rightarrow$ pt $P = \left(\frac{40}{13}, 0, \frac{-31}{13}\right)$

$\therefore$ The value of $26 \cdot a\,(PA)^2$

$= 26 \times 3 \times \left(\underbrace{\sqrt{\left(\frac{40}{13}-1\right)^2 + (0-1)^2 + \left(\frac{-31}{13}+1\right)^2}}_{\text{on simplification}}\right)^2$

$= 26 \times 3 \times \frac{468}{169}$

$= \boxed{216}$

Ans

To fill in the blanks

Q.25 Let $\vec{c}$ be the projection vector of $\vec{b} = \lambda\hat{\imath} + 4\hat{k}, \lambda > 0$, on the vector $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 2\hat{k}$. If $|\vec{a} + \vec{c}| = 7$, then the area of the parallelogram formed by the vectors $\vec{b}$ and $\vec{c}$ is ____________.

(JEE-Main 2025)

CONCEPT APPLICABLE AND SOLUTION:

→ Scalar projection of $\vec{a}$ on $\vec{b}$ $= \dfrac{\text{Dot Product of two given vector}}{\text{magnitude of on vector}}$

Vector Projection = (Scalar projection)(Unit vector in that direction)

i.e. $\left(\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}\right)\dfrac{\vec{b}}{|\vec{b}|} = \left(\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\right)\vec{b}$

ATQ

$$\vec{c} = \left(\frac{(\lambda\hat{i}+4\hat{k})\cdot(\hat{i}+2\hat{j}+2\hat{k})}{(1+4+4)}\right)(\hat{i}+2\hat{j}+2\hat{k})$$

$$\vec{c} = \left(\frac{\lambda+8}{9}\right)(\hat{i}+2\hat{j}+2\hat{k}) \quad \text{———— (A)}$$

Also ATQ

$$|\vec{a}+\vec{c}| = 7$$

$$\left|\vec{a} + \frac{\lambda+8}{9}\vec{a}\right| = 7$$

$$|\vec{a}|\left|\frac{\lambda+17}{9}\right| = 7$$

$|\vec{a}| = \sqrt{1+4+4} = 3$

Next Page

FPP: ON025

$$\cancel{3}\left|\frac{\lambda+17}{\cancel{9}\,3}\right| = 7$$

$$|\lambda+17| = 21$$

$$\lambda+17 = \pm 21$$

$$\lambda = +21-17 = 4$$

$\lambda = -21-17 = -38$ rejected as $\lambda = 0$

$$\therefore \boxed{\lambda = 4}$$

Subs vall of λ in eqn (A)

$$\vec{c} = \left(\frac{4+8}{9}\right)\left(\hat{i}+2\hat{j}+2\hat{k}\right)$$

$$\boxed{\vec{c} = \frac{4}{3}\left(\hat{i}+2\hat{j}+2\hat{k}\right)}$$

$$\vec{b} = 4\hat{i}+4\hat{k} = 4(\hat{i}+\hat{k})$$

Area of ||gm $= |\vec{b}\times\vec{c}|$

$$\vec{b}\times\vec{c} = 4\times\frac{4}{3}\begin{vmatrix} i & j & k \\ 1 & 0 & 1 \\ 1 & 2 & 2 \end{vmatrix} = \frac{16}{3}\left(-2\hat{i}-\hat{j}+2\hat{k}\right)$$

$\therefore$ Area $= |\vec{b}\times\vec{c}| = \frac{16}{\cancel{3}}\times\cancel{3}$

$= \boxed{16 \text{ sq units}}$

is ans to fill in the blanks