❖ Electric Current and Ohm's Law

If we maintain a constant potential difference between the two conductors, we get a constant flow of charge in a metallic wire connecting the two conductors. This flow of charge in metallic wire constitutes **electric current.**

The branch of Physics which deals with the charges in motion is called current electricity.

• Electric Current

It is defined as the rate of flow of electric charge through any cross-section of a conductor. It is denoted by *I*.

Electric current can be expressed by

$$\boxed{I = \frac{\text{Total charge flowing } (q)}{\text{Time taken } (t)} = \frac{ne}{t}}$$

where, n= number of free electrons

and e = electronic charge

Let Δq be the net charge flowing through a cross-section of the conductor in a particular direction during the time interval Δt [i.e. between times t and $(t + \Delta t)$] .

Then, at time t, the current in the conductor is given by

$$\boxed{I = \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t} = \frac{dq}{dt}}$$

It means that. the current through a conductor at a time is defined as the first derivative off charge passing through a cross-section of the conductor in a particular direction with respect to time.

Conventionally, the direction of **electric current is along the direction of motion of positive charges and opposite to the direction of motion of negative charges.**

- **Current is a scalar quantity.** Although it represents the direction of flow of positive charges, it does not follow the laws of vector addition (since the angle between the wires carrying current does not affect the total current). It follows the laws of scalar addition.
- **Unit of Electric Current**

CH3: CURRENT ELECTRICITY

The SI unit of current is ampere and it is represented by A.

1 ampere (A) = $\frac{1\ coulomb\ (C)}{1\ second\ (s)}$ =1 coulomb per second or 1 Cs^{-1}.

Current through a wire is said to be one ampere, if a charge of one coulomb flows through any cross-section of the wire in one second.

✓ Important Points

- As a matter of convention, the direction of flow of positive charge gives the direction of current. This is called **conventional current.**
- The direction of flow of electrons is opposite to the direction of electronic current. Therefore, the direction of electronic current is opposite to that of conventional current.
- Through a cross-section of the conductor in a time t, if a positive charge $q1$ is flowing from A to B and a negative charge q_2 is flowing from B to A, then total Current through the conductor is given by

$$I = \frac{q_1}{t} + \frac{q_2}{t} = \frac{q_1 + q_2}{t}$$

- The electric current which flows during the lightning, is of the order of tens of thousands of amperes. However, the current in our nerves is in microamperes.

• Current Density

The current density at a point in a conductor is defined as the amount of current flowing normally through unit area around that point of conductor.

If a current I is distributed uniformly over the cross-section area A of a conductor, then the current density at that point is

$$\boxed{J = \frac{I}{A}}$$

It is a characteristic property of a point inside the conductor. It is a vector quantity. Its direction at a point is the direction of flow of positive charge at that point.

The SI unit of current density is Am^{-2}.

❖ Electric Current in Conductors

All metals are good conductors of electricity. The electric conduction in them can be explained by the electron theory. In an atom of a substance, the electrons in the orbits close to the nucleus are bound to it due to the strong attraction of the positive nuclear charge, but the electrons far from the nucleus experience a very weak attractive force.

Hence, the outer electrons can be removed easily from the atom (by rubbing or by heating the substance). In fact, a few outer electrons leave their atoms and move freely within the substance (in the vacant spaces between the atoms). These electrons are called **free electrons or conduction electrons.**

They carry the charge in the substance from one place to the other. Therefore, the electrical conductivity of a solid substance depends upon the number of free electrons in it. In metals, this number is quite large ($\approx 10^{29}$ $/m^3$), Hence, **metals are good conductors of electricity**. Silver is the best conductor of electricity. It is better than copper, gold and aluminium.

In liquids and gases, electric conduction takes place by the movement of both positive and negative charge carriers ions whereas in metals, the electric conduction occurs by the movement of negative charge carriers (electrons) only.

The materials which have large number of free electrons and develop strong electric current, when electric field is applied on them are called conductors. Metals are good examples of conductors.

There are some other materials in which the electrons will be bound and they will not be accelerated, even if the electric field is applied, i.e. no current flows on applying electric field. Such materials are called insulators. e.g. wood, plastic, rubber, etc.

In our discussions, we will focus only on solid conductors in which the positive ions are at fixed positions and the current is carried by the negatively charged electrons.

❖ Flow of Electric Charge

(In difference of external electric field)

- When no electric field is applied on a solid conductor, the free electrons in them move like molecules in a gas due to their thermal velocities. There is no preferential direction for the velocities of the electrons. The average thermal random velocity is zero. Due to this, there is no net flow of electric charge in a particular direction inside the conductor and hence no current flows in it.
- When an electric field is applied on a solid conductor in the shape of a cylinder of circular cross-section by attaching positively and negatively charged circular discs of a dielectric of the same radius as that of the solid conductor, at the two ends, an electric field is generated in the conductor from positive charged disc towards negative charged disc.
- Electrolytic solutions are conductors of different types in which positive and negative charges can move.

❖ Ohm's Law

It states that the current I flowing through a conductor is directly proportional to the potential difference V across the ends of the conductor, provided that the physical conditions (temperature, mechanical strain, etc.) are kept constant. Mathematically,

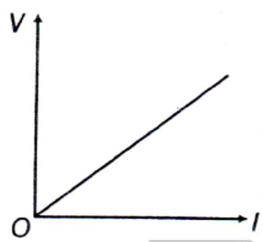
$$I \propto V \quad \text{or} \quad V \propto I \quad \text{or} \quad \boxed{V = IR}$$

or $$\boxed{\frac{V}{I} = R = \text{constant}}$$

where, R is resistance of the conductor.

Its value depends on the length, shape and the nature of the material of the conductor. It is independent of the values of V and I. Such conductors are said to obey Ohm's law.

For an ohmic conductor, the graph between V and I is a straight line as shown in the figure.



❖ Resistance of a Conductor

It is defined as the ratio of the potential difference applied across the ends of the conductor to the current flowing through it.

Mathematically, R= $\frac{V}{I}$

The SI unit of resistance is ohm and denoted by Ω

$$1 \text{ ohm } (\Omega) = \frac{1 \text{ volt (V)}}{1 \text{ ampere (A)}} = 1 \text{ volt/ampere (or V/A)}$$

The resistance of a conductor is said to be one ohm, if one ampere of current flows, when a potential difference of one volt is applied across the ends of the conductor.

Dimensional formula of electrical resistance is $[ML^2T^{-3}A^{-2}]$.

(i)
(ii) It is directly proportional to the length of the conductor.

i.e. $R \propto l$ ...(i)

(ii) It is inversely proportional to the area of the cross-section of the conductor.

i.e $R \propto \frac{1}{A}$ ...(ii)

From Eqs. (i) and (ii), we get

$$R \propto \frac{l}{A} \quad \text{or} \quad \boxed{R = \rho\frac{l}{A}}$$

where, ρ is the constant of proportionality known as resistivity or specific resistance of the conductor.

(iii) It depends on the nature of the material and temperature of the conductor.

Effect of Temperature on Resistance

Resistance of a metallic conductor at temperature t°C is given as

$$\boxed{R_t = R_0\,(1+\alpha t+\beta t^2)} \quad \text{...(i)}$$

where, R_0 = resistance of conductor at 0°C and α, ß are the temperature coefficients of resistance.

If the temperature t°C is not sufficiently large which is so in most of the practical cases, then Eq. (i) can be written as

$$R_t = R_0(1+\alpha t) \text{ or } \boxed{\alpha = \frac{R_t - R_0}{R_0 \times t}}$$

$$= \frac{\text{Change in resistance}}{\text{Original resistance} \times \text{Rise of temperature}}$$

Therefore, **temperature coefficient of resistance is defined as the increase in resistance per unit original resistance per degree Celsius or kelvin rise of temperature.**

For metals like silver, copper, etc., the value of α is positive, therefore, **resistance of the metal increases with the rise in temperature.**

For insulators and semiconductors, the value of α is negative, therefore **resistance decreases with the rise in temperature.**

For alloys like manganin, constantan, etc., the value of α is very small as compared to metals. The value of α is different at different temperatures.

Temperature coefficient of resistance averaged over the temperature range t_1°C to t_2°C is given by

$$\boxed{\alpha = \frac{R_2 - R_1}{R_1(t_2 - t_1)}}$$

where, R_1 and R_2 are the resistances at t_1°C to t_2°C respectively.

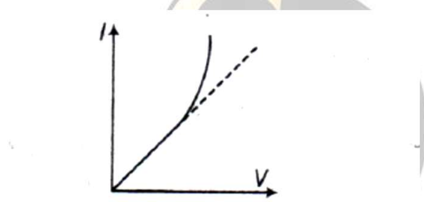
Due to high resistivity and low temperature coefficient, alloys are used to make standard resistance coils.

- ## Limitations of Ohm's Law

The devices which do not obey Ohm's law are called non-ohmic devices, such as vacuum tubes, semiconductor diodes, transistors, etc. The relation $\left(\frac{V}{I} = R\right)$ valid for both, ohmic and non-ohmic devices. **non-ohmic devices**, value of R is not constant. **For non-ohmic devices,** value of R is not constant i.e. Ohm’s law fails.

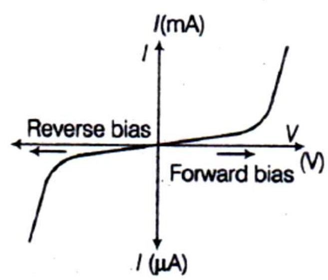
The limitations of Ohm's law are given below.

(i) Potential difference may vary non-linearly with Current.



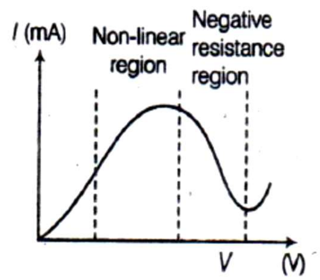
Variation of potential difference with current

(ii) The variation of current with potential difference may depend on the sign of potential difference applied.



Variation of current according to the sign of potential difference

(iii) The relation between potential difference and current is not unique, i.e. there is more than one value of V for the same current I.



Graph of current *versus* potential difference for GaAs

❖ Drift of Electrons and The Origin of Resistivity

- **Drift Velocity**

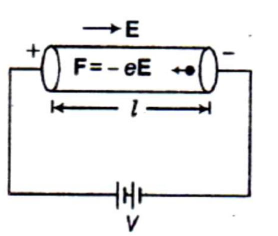
It is defined as the average velocity with which the free electrons in a conductor get drifted towards the positive end of the conductor under the influence of an electric field applied across the conductor.

It is denoted by v_d. The drift velocity of electron is of the order of $10^{-4} ms^{-1}$.

Let V be the potential difference applied across the ends of a conductor of length, then the magnitude of electric field is

$$\boxed{E = \frac{V}{l}}$$

The direction of electric field is from positive end to negative end of the conductor as shown in figure.



Current in a metallic conductor

Since, the charge on an electron is $-e$ and each free electron in the conductor experiences a force F.

$\therefore$ $$\boxed{\mathbf{F} = -e\mathbf{E}}$$

Here, negative sign indicates that the direction of force is opposite to that of electric field applied.

Acceleration of each electron is given by

$$\boxed{\mathbf{a} = -\frac{e\mathbf{E}}{m}}$$

$$\left[\text{from Newton's second law, } a = \frac{F}{m}\right]$$

where, m is mass of the electron.

Under the effect of applied electric field, the free electrons accelerate and acquire a velocity component in a direction opposite to the direction of electric field in addition to their thermal velocities.

However, this gain in velocity of electron due to the electric field is very small and it is lost in the next collision with the ion/atom of the conductor.

At any instant of time, the velocity acquired by electron having thermal velocity u, is given by

$$V_1 = u_1 + at_1$$

where, t_1 is the time elapsed as it has suffered its last collision with ion or atom of conductor.

Similarly, $\mathbf{v}_2 = \mathbf{u}_2 + \mathbf{a}t_2, \ldots, \mathbf{v}_n = \mathbf{u}_n + \mathbf{a}t_n$

$$\therefore \text{ Average velocity, } \mathbf{v}_d = \frac{\mathbf{v}_1 + \mathbf{v}_2 + \ldots + \mathbf{v}_n}{n}$$

$$= \frac{(\mathbf{u}_1 + \mathbf{a}\,t_1) + (\mathbf{u}_2 + \mathbf{a}\,t_2) + \cdots + (\mathbf{u}_n + \mathbf{a}\,t_n)}{n}$$

$$= \frac{\mathbf{u}_1 + \mathbf{u}_2 + \ldots + \mathbf{u}_n}{n} + \frac{\mathbf{a}[t_1 + t_2 + \ldots + t_n]}{n}$$

We know that, $\frac{\mathbf{u}_1 + \mathbf{u}_2 + \ldots + \mathbf{u}_n}{n} = 0$ and $\frac{t_1 + t_2 + \ldots + t_n}{n}$ is called average time elapsed or average relaxation time and is denoted by τ. Its value is of the order of 10^{-14} s.

$$\therefore \quad \mathbf{v}_d = 0 + \mathbf{a}\tau$$

$$\text{or} \quad \mathbf{v}_d = \mathbf{a}\tau$$

$$\text{or} \quad \boxed{\mathbf{v}_d = -\frac{e\mathbf{E}}{m}\tau}$$

Negative sign shows that v_d is opposite to the direction of E

$$\therefore \quad \boxed{\text{Average drift velocity, } \mathbf{v}_d = \frac{e\mathbf{E}}{m}\tau}$$

Average relaxation time = Mean free path of electron/drift speed of electron.

- **Relation between Drift Velocity and Electric Current**

Consider a conductor of length l and A be the uniform area of cross-section.

$\therefore$ Volume of conductor$= Al$

If the conductor contains n free electrons per unit volume, then number of free electrons in the conductor = nAl .

If e is the charge of an electron, then total charge on all free electrons in the conductor is given by

$$q = nAle \quad \text{...(i)}$$

Time taken by the free electrons to cross the length of the conductor

$$t = \frac{l}{v_d} \quad \text{...(ii)}$$

Since, current is the rate of flow of the charge through conductor.

$$l = \frac{q}{t}$$

Using Eqs. (i) and (ii), we get

$$I = \frac{nAle}{l/v_d}$$

$$\Rightarrow \quad \boxed{I = n\,eA\,v_d} \quad \text{...(iii)}$$

Eq. (iii) gives the relation between the current flowing through the conductor and drift velocity of the electron.

Putting the value of $\left(v_d = \frac{eE\tau}{m}\right)$ in Eq. (iii), we get $\boxed{I = \frac{Ane^2\tau E}{m}}$

- **Deduction of Ohm's Law**

We know that, $I = \frac{Ane^2\tau E}{m}$ and $v_d = \frac{eE}{m}.\tau = \left(\frac{eV}{ml}\tau\right)$ $\left[\text{as } E = \frac{V}{l}\right]$

$$I = Anev_d = Ane\left(\frac{eV}{ml}\tau\right) = \left(\frac{Ane^2\tau}{ml}\right)V$$

rn

So,

or $$\frac{V}{I} = \frac{ml}{Ane^2\tau} = R = \text{constant}$$

$$\therefore \quad \frac{V}{I} = R \Rightarrow \boxed{V = RI}$$

This is called Ohm's law.

❖ Mobility

It is defined as the magnitude of drift velocity of charge per unit electric field applied. It is expressed as

$$\mu = \frac{\text{Drift velocity } (v_d)}{\text{Electric field } (E)} = \frac{qE\tau/m}{E} = \frac{q\tau}{m}$$

where, τ is the average relaxation time of the charge while drifting towards the opposite electrode and m is the mass of the charged particle.

Mobility of electrons, $\mu_e = \dfrac{e\tau_e}{m_e}$.

and mobility of holes, $\mu_h = \dfrac{e\tau_h}{m_h}$.

where, τ_e and τ_h are average relaxation time for electrons and holes, respectively. m_e and m_h refer to mass of electrons and holes, respectively (either electron or hole).

The SI unit of mobility is $m^2s^{-1}V^{-1}$ or $m\ s^{-1}N^{-1}C$ and it is in the order of 10^4 in practical units ($cm^2s^{-1}V^{-1}$).

Mobility is positive for both positive current carriers and negative current carriers.

The total current in the conducting material is the sum of the currents due to the positive current carriers (holes) and negative current carriers (electrons).

❖ Resistivity of Various Materials

Specific resistance or resistivity of the material of a conductor is defined as the resistance of a unit length with unit area of cross-section of the material of the conductor, i.e. it is also defined as the resistance of unit cube of a material of the given conductor.

The SI unit of resistivity is ohm-metre or Ω-m and its dimensional formula is $[ML^3T^{-3}A^{-2}]$.

Since, we know that $R = \rho \frac{l}{A}$

$\Rightarrow$ $\rho = \frac{RA}{l}$ ...(i)

Substituting the value of $R = \frac{ml}{ne^2 A\tau}$ in Eq. (i),

We have $\rho = \left(\frac{ml}{ne^2 A\tau}\right) \cdot \frac{A}{l}$

∴ Resistivity of the material, $\boxed{\rho = \frac{m}{ne^2\tau}}$

From the above formula, it is clear that resistivity of a conductor depends upon the following factors

(i) $\rho \propto \frac{1}{n}$, i.e. resistivity of material is inversely proportional to the number density of free electrons (number of free electrons per unit volume).

As the free electron density depends on the nature of material, so resistivity of a conductor depends on the nature of the material.

(ii) $\rho \propto 1/\tau$, i.e. resistivity of a material is inversely proportional to the average relaxation time τ of free electrons in the conductor.

As value of t depends on the temperature of conductor, so resistivity of a conductor changes with temperature and as temperature increases, τ decreases, hence ρ increases.

✓ **Important:**

As electric resistivity of copper is low and its electrical conductivity is high, so due to this reason, copper wire is used as connecting wire.

- **Temperature Dependence of Resistivity**

Resistivity of a metal conductor is given by

$$\rho = \rho_0[1 + \alpha(T - T_0)] \qquad \text{...(i)}$$

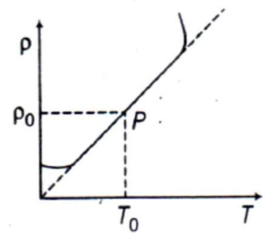
Where ρ = resistivity at temperature T

and ρo = resistivity at temperature T_0

$$\Rightarrow \quad \boxed{\alpha = \frac{\rho - \rho_0}{\rho_0 (T - T_0)} = \frac{d\rho}{\rho_0} \cdot \frac{1}{dT}}$$

Thus, temperature coefficient of electrical resistivity is also defined as the fractional change in electrical resistivity $\frac{d\rho}{\rho o}$ per unit change in temperature dT.

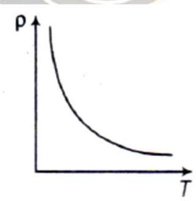
- For metals, the value of α is positive, therefore resistivity of metal increases with increase in temperature.



Resistivity as a function of temperature for metals

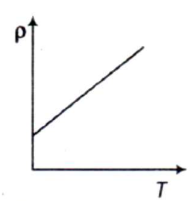
Eq.(i) implies that a graph of ρ plotted against T would be a straight line. At temperatures much lower than 0°C, the graph deviates considerably from a straight line. Eq.(i) can be used approximately over a limited range of T around any reference temperature T_o, where the graph can be approximated as a straight line.

- For semiconductors, the resistivity decreases with increase in temperature.



Resistivity as a function of temperature for semiconductors

- For alloys, the resistivity is very large, but has weak dependence on temperature.



Resistivity as a function of temperature for alloys

✓ **Important**

Electric fuse is made of an alloy of zinc, copper, silver and aluminium. This is because alloys have low resistivity. This causes the wire to melt, if a current more than safe current flows through the circuit.

Conductance and Conductivity

Conductance

It is defined as the reciprocal of resistance of a conductor. It is expressed as

$$\boxed{G = \frac{1}{R}}$$

Its SI unit is mho (Ω^{-1}) or siemen (S).

The dimensional formula of conductance is [$M^{-1}L^{-2}T^{3}A^{2}$].

Conductivity

It is defined as the reciprocal of resistivity of a conductor. It is expressed as

$$\boxed{\sigma = \frac{1}{\rho}}$$

The SI unit is mho per metre ($\Omega^{-1}m^{-1}$) or siemen per meter (S/m)

The dimensional formula of conductivity is ($M^{-1}L^{-3}T^{3}A^{2}$].

Relation between J, G and E

Since, the relation between the current flowing through the conductor and drift velocity of electron is given by

$$I = nAe\,v_d$$

$$\therefore \quad I = nAe\left(\frac{eE}{m}\tau\right) = \frac{nAe^2\tau E}{m} \quad \left\{\because v_d = \frac{eE}{m}\tau\right\}$$

$$\Rightarrow \quad \frac{I}{A} = \frac{ne^2\tau E}{m} \Rightarrow J = \frac{ne^2\tau E}{m} \quad \left[\because J = \frac{I}{A}\right]$$

$$\text{or} \quad J = \frac{1}{\rho}E \quad \left[\because \rho = \frac{m}{ne^2\tau}\right]$$

$$\therefore \quad \boxed{J = \sigma E} \quad \left[\because \sigma = \frac{1}{\rho}\right]$$

It is a microscopic form of Ohm's law.

❖ Classification of Materials in Terms of Conductivity

On the basis of conductivity, the materials can be classified into the following categories

Insulators Those materials whose electrical conductivity is either very small or nil, are called insulators e.g. glass, rubber, etc.

Conductors Those materials whose electrical conductivity is very high are called conductors e.g. silver, aluminium, etc.

Semiconductors Those materials whose electrical conductivity lies in between that of insulators and conductors are called semiconductors e.g. germanium, silicon, etc.

- **Thermistor** A thermistor is a heat sensitive device whose resistivity changes very rapidly with change of temperature.

  A thermistor can have a resistance in the range of 0.1 Ω to 10^7 Ω depending upon its composition.

- **Superconductivity** The resistivity of certain metal or alloy drops to zero, when they are cooled below a certain temperature is called superconductivity. It was observed by Prof. Kamerlingh in 1911.

❖ Electrical Energy

- ### Heating Effect of Electric Current

When an electric current is passed through a conductor, then it becomes hot after sometime. This effect is called heating effect of current or **Joule's heating effect.**

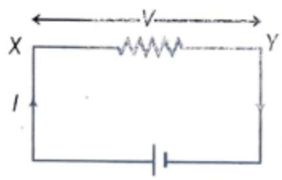
It is used to form various electrical appliances such as electric bulb, electric furnace, geyser, electric press, etc.

Joule in 1941 studied experimentally the heating effect of current and found that the amount of heat (H)produced, when current (I) flows through a conductor of resistance (R) for time (t) is given by

$$\boxed{H = I^2Rt \text{ (in joule)}}$$

- **Heat Produced by Electric Current**

Consider a conductor XY of resistance R. Let V be the potential difference applied across ends of XY.



I = Current flowing through XY

t = time for which the current is flowing

∴ Total charge flowing from X to Y, $q = It$

∴ Work done in carrying a charge from X to Y is

$$W = V X q = VIt \; joule$$

$$V = IR = W = I^2 \, Rt \; joule$$

If this electric work done appears as heat, then the amount of heat produced,

$$\boxed{H = W = I^2Rt \text{ joule} = \frac{I^2Rt}{4.2} \text{ cal}}$$

This is known as joule's law of heating.

- **Different Aspects of Heating Effects of Current**

(i) The wire of heating element of heavy appliances such as electric iron, electric heater, heating rod, etc is made up of nichrome. It is because

(a) it has high melting point and high value of resistance,

(b) it is not oxidised easily when heated in air.

(ii) A fuse wire is generally made of tin-lead alloy, because it has low resistivity, low melting point and suitable current rating corresponding to load in the circuit.

❖ Electrical Energy and Power

• Electrical Energy

It is defined as the total work done W by the source of emf V in maintaining the electric current I in the given circuit for a specified time t.

According to Ohm's law, we have, $V = IR$

Total charge that crosses the resistor is given by q = It Energy gained is given by

$$E = W = Vq$$
$$= V\,(It) = VIt$$
$$= [IR]\,It = I^2 R\,t \qquad [\because V = IR]$$
$$= \left[\frac{V}{R}\right]^2 Rt = \frac{V^2 t}{R} \qquad \left[\because I = \frac{V}{R}\right]$$

$$\therefore \quad \boxed{E = VIt = I^2 Rt = \frac{V^2}{R} t}$$

The SI unit of electrical energy is joule (J),

where, 1 joule = 1 volt x 1 ampere X 1 s = I watt x ls

• Commercial Unit of Electrical Energy

To measure the electrical energy consumed commercially, the unit of energy, i.e. joule is no sufficient. So, to express electrical energy consumed commercially, a special unit called kilowatt hour (kWh) is used in place of joule.

1kWh is also called 1 unit of electrical energy or Board of Trade unit' (BoT).

1 kilowatt hour or 1 unit of electrical energy is the amount of energy dissipated in 1 hour in a circuit, when the electric power in the circuit is 1 kilowatt.

1 kilowatt hour (kWh) =1 kilowatt x hour= 3.6 x 10^6 joule (J)

• Electrical Power

It is defined as the rate of electrical energy supplied per unit time to maintain the flow of electric current through a conductor.

Mathematically, $\boxed{P = \frac{W}{t} = VI = I^2R = \frac{V^2}{R}}$

$[\because\ W = VIt]$

The SI unit of power is watt (W).

Power of an electric circuit is said to be one watt, if one ampere current flows in it against a potential difference of one volt. The bigger units of electrical power are kilowatt (kW) and megawatt (MW).

where, 1 kW= 1000 W and 1 MW = $10^6 W$

Commercial unit of electrical power is horse power (HP),

where,1HP = 746 W.

Cells, EMF and Internal Resistance

Cells

An electric cell is a source of energy that maintains a continuous flow of charge in a circuit. Electric cell converts chemical energy into electrical energy.

Electromotive Force (EME) of a Cell (E)

Electric cell has to do some work in maintaining the current through a circuit. The work done by the cell in moving unit positive charge through the whole circuit (including the cell) is called the electromotive force (emf) of the cell.

If during the flow of q coulomb of charge in an electric circuit, the work done by the cell is W, then

emf of the cell, $E = \frac{W}{q}$

Its unit is joule/coulomb or volt.

If W=1 joule and q= l coulomb, then E=1 volt, i.e. if in the flow of 1 coulomb of charge, the work done by the cell is 1 joule, then the emf of the cell is 1 volt.

Internal Resistance (r)

Internal resistance of a cell is defined as the resistance offered by the electrolyte and electrodes of the cell to the flow of current through it. It is denoted by r. Its unit is ohm.

Internal resistance of a cell depends on the following factors

(i) It is directly proportional to the separation between the two plates of the cell.
(ii) It is inversely proportional to the area of plate dipped into the electrolyte.
(iii) It depends on the nature, concentration and temperature of the electrolyte and increases with increase in concentration.

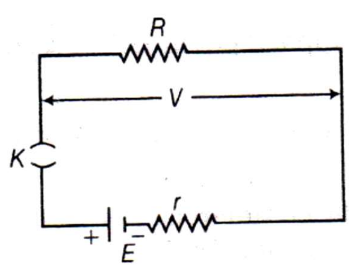
Terminal Potential Difference(V)

Terminal potential difference of a cell is defined as the potential difference between the two terminals of the cell in a closed circuit (i.e. when current is drawn from the cell). It is represented by V and its unit is volt.

Thus, terminal potential difference of a cell is always less than the emf of the cell. In a closed circuit, there is some fall in emf of the cell due to current flow through its internal resistance.

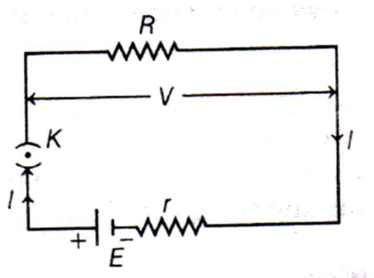
Relation between Terminal Potential Difference, emf of a Cell and Internal Resistance of a Cell

(i) If no current is drawn from the cell, i.e. the cell is in open circuit, so emf of the cell will be equal to the terminal potential difference of the cell.



$I = 0 \quad \text{or} \quad V = E$

(ii) Consider a cell of emf E and internal resistance r is connected across an external resistance R.



Current drawn from the cell, $I = \frac{E}{R+r}$ ...(i)

Where, $E =$ emf of the cell,

$R =$ External resistance

And $r =$ internal resistance of a cell.

Now, from Ohm's law, $V = IR$

$\Rightarrow$ $I = \frac{V}{R}$ ...(ii)

From Eqs. (i) and (ii), we get

$$\frac{V}{R} = \frac{E}{R+r}$$

$$\Rightarrow \boxed{r = \left(\frac{E}{V} - 1\right)R}$$

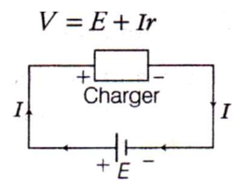
From definition of terminal potential difference,

$$\boxed{V = E - Ir}$$

Charging of a Cell

- During charging of a cell, the positive terminal (electrode) of the cell is connected to positive terminal of battery charger and negative terminal (electrode) of the cell is connected to negative terminal of battery charger. In this process, current flows from positive electrode to negative electrode of the cell. From the given figure,

$$V = E + Ir$$



- Thus, the terminal potential difference of a cell becomes greater than the emf of the cell.
- The potential drop across internal resistance of the cell is called lost voltage, as it is not indicated by a voltmeter. Its value is equal to Ir.

Difference between EMF and Potential Difference of a Cell

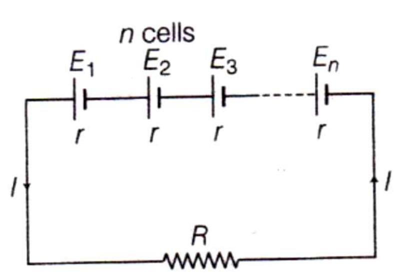
EMF	Terminal potential difference
The emf of a cell is the maximum potential difference between the two electrodes (terminals) of a cell, when the cell is in the open circuit.	The Terminal potential difference The terminal potential difference of a cell is the potential difference between the two terminals of the cell in a closed circuit.
It is independent of the resistance of the circuit and depends upon the nature of electrodes and electrolyte of the cell.	It depends upon the resistance of the circuit and current flowing through it.
The term emf is used for the source of electric current.	The potential difference is measured between any two points of the electric circuit.
The emf is a cause.	The potential difference is an effect.

❖ Cells in Series and Parallel

✓ Cells in Series

When the negative terminal of one cell is connected to the positive terminal of the other cell, then this combination of cells is called series combination.

Let n identical cells each of emf E and internal resistance r be connected in series to the external resistance Ras shown in the following figure.



✓ **Important Points:**

(i) The equivalent emf of a series combination of n cells is equal to the sum of their individual emfs.

(ii) The equivalent internal resistance of a series combination of n cells is equal to sum of their individual internal resistances.

Equivalent emf of n cells in series,

$E\text{eq} = E_1 + E_2 + \ldots +$ upto n terms $= nE$

Equivalent emf of n cells in series,

$r\text{eq} = r_1 + r_2 + \ldots +$ upto n terms $= nr$

Equivalent internal resistance of n cells in series,

Total resistance of the circuit $nr + R$

∴ Current in the resistance R is given by

$$\boxed{I = \frac{nE}{R + nr}}$$

where, n = number of cells,

r = internal resistance, r

R = external resistance,

E = emf of cell.

and I = current flowing

Case I When R<< $nr, then$

$I = \frac{E}{r}$ = current due to a single cell

Case II When R>>$nr, then$

$I = \frac{nE}{R} = n$ times the current due to a single cell = maximum current

Case II When cells are of same emf and different internal resistances, then

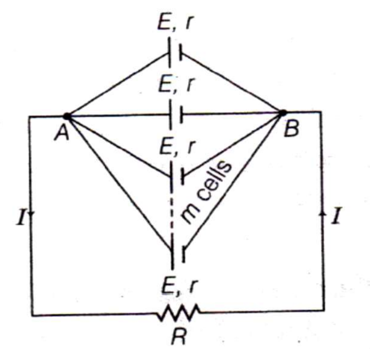
$$I = \frac{E_1 + E_2 + \ldots + E_n}{R + (r_1 + r_2 + \ldots + r_n)}.$$

✓The maximum current can be drawn from the parallel combination of cells, if the external resistance is very low as compared to the total internal resistance of the cells.

❖ Calls in Parallel

When the positive terminals of all the cells are connected to one point and the negative terminals are connected to other point, then this combination of cells is called parallel combination.

Let m cells each of emf E and internal resistance r be connected in parallel with the external resistance R as shown in the following figure.



✓**Important:**

(i) The equivalent emf of parallel combination of cells of same emfs is equal of one cell.

(ii) The reciprocal of equivalent internal resistance of parallel combination of cells is equal to the sum of the reciprocals of the internal resistance of each cell.

$\therefore \quad \frac{1}{r_p} = \frac{1}{r_1} + \frac{1}{r_2} + \ldots$ upto m terms $= \frac{m}{r}$ or $r_p = \frac{r}{m}$

As, R and r_{p} are in series, so total resistance in the circuit $= R + \frac{r}{m}$

In parallel combination of identical calls, the effective emf in the circuit is equal to the emf due to a single cell, because in this combination only the size of the electrodes increases but not emf.

$\therefore$ Current in the resistance R is given by

$$\boxed{I = \frac{E}{R + \frac{r}{m}}}$$

Case I When $R \gg \frac{r}{m}$, then

$I = \frac{E}{R}$ = current due to a single cell

Case II When $R \ll \frac{r}{m}$, then $I = \frac{E}{r/m}$

$= \frac{mE}{r}$ = m times current due to a single cell = maximum current

Case III When cells are of same emf and different internal resistances, then

$$I = \frac{E}{R + r'} \qquad [\because E_1 = E_2 = \ldots E_n = E]$$

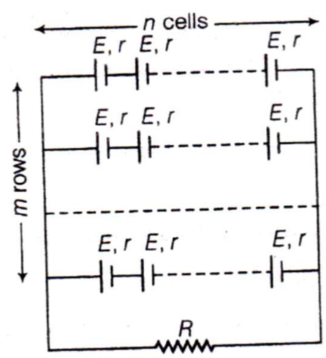
where, $\frac{1}{r'} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \ldots + \frac{1}{r_n}$ and E is emf of each cell.

Note The maximum current can be drawn from the parallel combination of cells, if the external resistance is very low as compared to the total internal resistance of the cells.

❖ Mixed Combination of Cells

When a certain number of cells are connected in various series and all such series are mutually connected in parallel, then this combination of cells is called mixed combination. Let there be n cells in series in one row and m rows of cells in parallel.

Suppose all the cells are identical. Let each cell be of emf E and internal resistance r.



Equivalent emf of each row = nE

Equivalent internal

resistance of each row $= nr$

Total emf of combination = nE

Total internal resistance of combination,

$$\frac{1}{r'} = \frac{1}{nr} + \frac{1}{nr} + \ldots \text{ upto } m \text{ times}$$

$$\frac{1}{r'} = \frac{m}{nr} \quad \text{or} \qquad r' = \frac{nr}{m}$$

Total resistance of the circuit $= r' + R = \frac{nr}{m} + R$

Current in the resistance R is given by

$$\boxed{I = \frac{nE}{\frac{nr}{m} + R}}$$

Thus, we get the maximum current in mixed grouping of cells, if the value of external resistance is equal to the total internal resistance of all the cells, i.e. external resistance = total internal resistance of all the cells

$$\left(R = \frac{nr}{m}\right).$$

❖ Combination of Two Cells in Series and Parallel (With Different emfs and Internal Resistances)

Two Cells in Series

The two cells are said to be connected in series between two points A and C, when negative terminal of one cell is connected to positive terminal of other cell as shown in the Fig. (a).

Let E_1, E_2 be the emfs of the two cells and r_1, r_2 be their internal resistances, respectively.

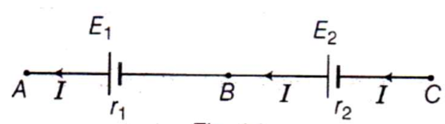


Fig. (a)

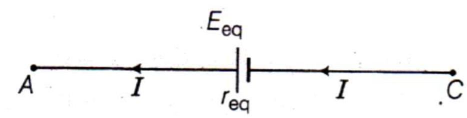


Fig. (b)

If n cells of emfs E_1, E_2 ... E_n and of internal resistances r_1, r_2, ... , r_n respectively, are connected in series between points A and C, then equivalent emf is given by

$$\boxed{E_{eq} = E_1 + E_2 + \ldots + E_n}$$

Equivalent internal resistance of the cells is given by

$$\boxed{r_{eq} = r_1 + r_2 + \ldots + r_n}$$

In the series combination of two cells, if negative terminal of first cell is connected to the negative terminal of the second cell between points A and C, as shown in the Fig. (c), then

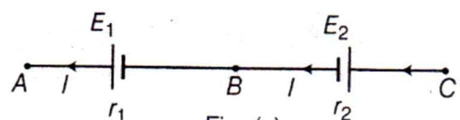


Fig. (c)

Then, equivalent emf of the two cells is $E_{eq} = E_1 - E_2$

But equivalent internal resistance is $r_{eq} = r_1 + r_2$.

❖ Two Cells in Parallel

The two cells are said to be connected in parallel between two points A and C, when positive terminal of each cell is connected to one point and negative terminal of each cell is connected to the other point as shown in the Fig. (a).

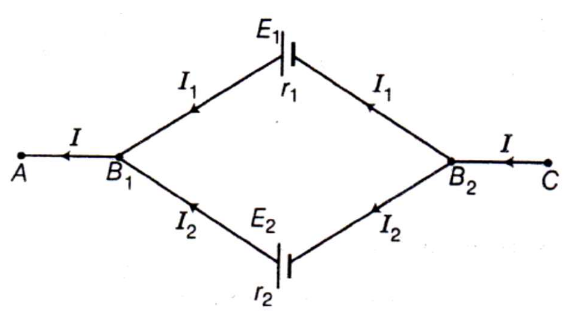


Fig.(a)

Fig.(b)

Let E_1, E_2 be the emfs of the two cells and r_1, r_2 be their internal resistances, respectively.

If n cells of emfs $E_1, E_2, \ldots, E_n$ and internal resistances $r_1, r_2, \ldots, r_n$ are connected in parallel, whose equivalent emf is E_{eq} and equivalent internal resistance is r_{eq}, then

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \ldots + \frac{1}{r_n}$$

and $$\frac{E_{eq}}{r_{eq}} = \frac{E_1}{r_1} + \frac{E_2}{r_2} + \ldots + \frac{E_n}{r_n}$$

If the two cells are connected in parallel and are of the same emf E and same internal resistance r, then

$$E_{eq} = \frac{Er + Er}{r + r} = E$$

$$\frac{1}{r_{eq}} = \frac{1}{r} + \frac{1}{r} = \frac{2}{r} \Rightarrow r_{eq} = \frac{r}{2}$$

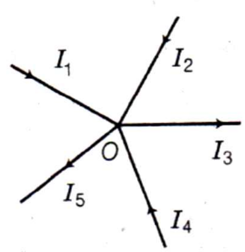
❖ Kirchhoff's Laws and its Applications

❖ Kirchhoff's Rules or Laws

- **First Law (Kirchhoff's Junction Rule)**

This law states that the algebraic sum of the currents meeting at a point in an electrical circuit is always zero. It is also known as Kirchhoff's junction rule or Kirchhoff's current rule. This rule is based on conservation of charge.

i.e. $$\boxed{\Sigma I = 0}$$



Electric junction

Consider a point O in an electrical circuit at which currents $I1$, $I2$, $I3$, $I4$ and $I5$ lowing through the different conductors meet, as shown in the above figure.

According to Kirchhoff 's first law, we have

$$I_1 + I_2 + (-I_3) + I_4 + (-I_5) = 0$$

$$\Rightarrow \quad I_1 + I_2 - I_3 + I_4 - I_5 = 0$$

$$\therefore \quad I_1 + I_2 + I_4 = I_3 + I_5$$

So, junction rule can also be stated as the sum of currents entering the junction is equal to the sum of currents leaving the junction, i.e. current cannot be stored at a junction.

- **Sign Convention for Kirchhoff's First Law**

The current flowing towards the junction of conductors is considered as positive and the current flowing away from the junction is taken as negative.

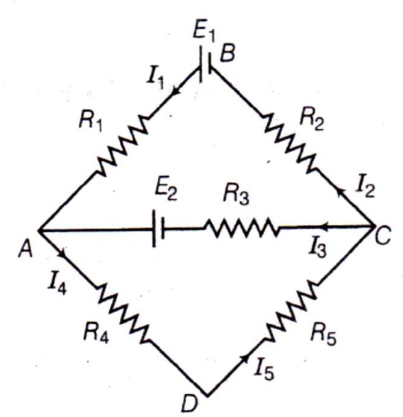
- **Second Law (Kirchhoff's Voltage Rule)**

This law states that the algebraic sum of changes in potential around any closed loop involving resistors and cells in the loop is zero.

i.e., $\boxed{\Sigma \Delta V = 0}$

It means that in any closed part of an electrical circuit, the algebraic sum of the emfs is equal to the algebraic sum of the products of the resistances and currents flowing through them. It is also known as **Kirchhoff's loop rule.**

Consider a closed electrical circuit $ABCDA$ containing two cells E_1 and E_2 and five resistances R_1, R_2, R_3, R_4 and R_5



Consider the closed loop $ABCA$. E_1, will send current in anti-clockwise and E_2 will send current in clockwise direction.

$\therefore$ Total emf of closed loop

$$ABCA = E_1 + (-E_2) = E_1 - E_2$$

But currents (I_1 and I_2) flow in anti-clockwise direction while current I_3 flows in clockwise direction.

The algebraic sum of products of resistances and Current

$$= I_1R_1 + I_2R_2 + (-I_3)\ R_3$$
$$= I_1R_1 + I_2R_2 - I_3R_3$$

∴ According to second law, for closed part $ABCA$,

$$E_1 - E_2 = I_1R_1 + I_2R_2 - I_3R_3$$

Similarly, for closed part $ACDA$

$$E_2 = I_3R_3 + I_4R_4 + I_5R_5$$

Thus, Kirchhoff's second law is based on law of conservation of energy.

- **Sign Convention for Kirchhoff 's Second Law**

The product of resistance and current in an arm of the loop is taken as positive, if the direction of current in that arm is in the opposite sense as one moves and is taken as negative, if the direction of current in an arm is the same sense as one moves.

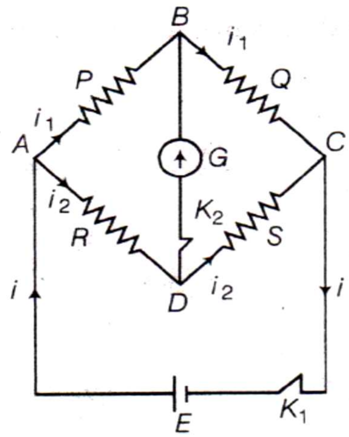
While traversing a loop, the emf of a cell is taken negative, if positive pole of the cell is encountered first, otherwise positive.

❖ Wheatstone Bridge

It is an arrangement of four resistances in a parallelogram which is used to measure one resistance in terms of the other three.

Consider four resistances P, Q, R and S are connected in the four arms of a parallelogram $ABCD$. The galvanometer G and a tapping key K_2 are connected between points B and D. The cell of emf E and one-way key K_1 are connected between points A and C as shown in the figure. Resistances P and Q are called ratio arms, resistance R is a variable resistance and S is unknown resistance.

The bridge is said to be balanced, when the galvanometer gives zero deflection, by adjusting the resistances of the four arms of parallelogram on sending current by the cell.



Wheatstone bridge

Thus, we have balance condition as

$$\boxed{\frac{P}{Q} = \frac{R}{S}}$$

Derivation:

In the given figure, let the current i be divided into two parts i_1 and i_2.

One part i_1, flows in the arm AB and the other part i_2, flows in the arm AD. The resistances P, Q, R and S are so adjusted that on pressing the key K_2, there is no deflection in the galvanometer G. That is, there is no current in the diagonal BD. Thus, the same current i_1 will flow in the arm BC as in the arm AB and the i_2 will flow in the arm DC as in the arm AD.

Applying Kirchhoff 's second law for the closed loop $BADB$, we have

$$-i_1P + i_2R = 0$$

$$Pi_1 = Ri_2 \qquad \text{... (i)}$$

Similarly, for the closed loop $CBDC$, we have

$$-i_1Q + i_2S = 0$$

$$Qi_1 = Si_2 \qquad \text{... (ii)}$$

Dividing Eq. (i) by Eq. (ii), we have

$$\frac{i_1P}{i_1Q} = \frac{i_2R}{i_2S}$$

or $$\frac{P}{Q} = \frac{R}{S}$$

It is clear from this formula that if the ratio of the resistances P and Q and resistance R are known, then the unknown resistance S can be calculated. This is why, the arms AB and BC are called ratio arms, arm AD known arm and arm CD unknown arm.

When the bridge is balanced, then on interchanging the positions of the galvanometer and the cell, there is no effect on the balance condition of the bridge.

Hence, the arms BD and AC are called conjugate arms of the bridge. (In balanced state, no current flows in the galvanometer arm, hence while computing, the equivalent resistance between A and C the resistance connected between B and D may be neglected.) The sensitivity of the bridge depends on the values of the resistance.

According to Maxwell, for greater sensitivity of the bridge, the galvanometer or the battery whichever has the higher resistance should be connected across the junctions of two highest and two lowest resistances.

The advantage of null point method/zero deflection in a Wheatstone bridge is that the resistance of galvanometer does not affect the balance point, there is no need to determine current in resistances and internal resistance of a galvanometer.

✓ **Point to be noted**:

- The Wheatstone bridge is most sensitive, when the resistance of all the four arms of the bridge is of same
- The Wheatstone bridge method is remains unaffected by the internal resistance of the source and resistance of the galvanometer
- The Wheatstone bridge method is a null method, hence the measurement of unknown resistance is very accurate
- The Wheatstone bridge can not be used to measure very high or very low resistance,

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CH3: CURRENT ELECTRICITY

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Syllabus latest as per NCERT:

Frame of reference, Motion in a straight line, elementary concepts of differentiation and integration for describing motion, uniform, and non-uniform motion, instantons velocity, uniformly accelerated motion, velocity-time and position -time graphs. Relations for uniformly accelerated motion (graphical treatment)

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